P-005

Estimating Uncertainty in Well Log Analysis by Monte Carlo Simulation

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Summary

Quantitative interpretation of well logs is always associated with some amount of uncertainty. The source of this uncertainty lies in errors inherent in the values of input parameters, uncertainties in tools response and errors in laboratory measurement. The above sources of errors are good reasons for the need for properly assessing the uncertainty in reservoir parameters obtained from well logs. A method to estimate this uncertainty by Monte Carlo simulation procedure has been illustrated.

Keyword: *Well Log Analysis*

Introduction

Quantitative analysis of well logs is based on measurements of resistivity, bulk density, interval transit time, self potential, natural radioactivity and hydrogen content of rock in a borehole. These measurements yield porosity, formation water resistivity, shale volume, water saturation and permeability through the use of different empirical relationships. These reservoir characteristics are then used to compute hydrocarbon volume and productivity.

One of the main problems associated with the evaluation of hydrocarbon volume and productivity of an oil or gas well is the amount of uncertainty that exists in the measurements made in a borehole and the errors inherent in subsequent computations. Formation heterogeneity errors, uncertainties in tools response, laboratory measurement errors and errors associated with other empirical constants create uncertainties that are often difficult to estimate at every depth interval in the borehole. In addition, errors in the measurements and the parameters used in the empirical relationships are carried forward and may not be accounted adequately in the final solution. When millions of dollars are at stake, everyone involved strives to understand the inherent risks in pursuing. A suitable and easily usable method is needed for quantifying the uncertainty in each of the measured and calculated parameters. Monte Carlo simulation is one such method that has been used as a tool to provide a method of quantifying uncertainty to the well log derived parameters.

Monte Carlo Simulation

Monte Carlo Simulation is a mathematical technique that allows one to estimate the uncertainty in quantitative analysis and decision making. The technique was first used by scientists working on the atom bomb.

The method consists in mathematically simulating an experiment to determine the probability distribution of a variable obtained from a mathematical or empirical relation which involves one or more input parameters, each of which has its associated uncertainty.

For implementing this method, each input parameter is regarded as a random variable. The probability distribution for a particular parameter is determined on the basis of known data. Different input parameters may have different statistical distributions (uniform, triangular, normal or log normal distribution) or one or more of these may have the same distribution. A large number of random numbers are generated that simulate the probability distribution of each parameter. The method allows computation of probability distribution of the output variable using the mathematical or empirical relationship. A different set of random numbers are generated for each input parameter even if these follow the same distribution. The computed probability distribution approximates the true probability distribution as if the process of the output variable being studied were conducted an infinite number of times. Approximate distribution approaches true distribution when the number of random numbers generated becomes very large.

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A set of well logs was used to identify an interval of interest and determine the type of probability distribution that the well log derived parameters follow. The available logs indicate that interval of interest is a gas bearing carbonate rock at a depth of 1660-1701 m.

Porosity (ϕ), resistivity of formation water (R_w) and true resistivity of formation (R_t) are the log derived quantities each of which is ultimately used in Archie"s equation to find water saturation (S_w).

$$S_w= \{F.R_w/R_t\}^n \qquad \text{...... (1)}$$

F is formation factor related to the porosity by the relation

$$F=a/\phi^m \qquad \text{......(2)}$$

where a, m and n are empirical constants referred as tortuosity factor, cementation factor and saturation exponent respectively.

R_t is determined from the deep laterolog (LLD). Effective porosity (Φ) for gas bearing formation is calculated using equation.

$$\Phi=\mathrm{sqrt}\{(\Phi_N{}^2+ \Phi^2{}_D) /2\} \qquad \text{......(3)}$$

where Φ_D and Φ_N are the density and neutron porosities respectively.

Density porosity, Φ_D is determined from the relationship

$$\Phi_D=(\rho_{ma}- \rho_b)/(\rho_{ma}- \rho_f) \qquad \text{........(4)}$$

where ρ_{ma} (=2.71 for calcite) and ρ_f (1.1 for saline water) is matrix and fluid densities respectively. Bulk density, ρ_b, is obtained from the density log.

R_w can be easily obtained from the SP log recorded in clean formation using the following equation

$$SSP= -K*\log(R_{mfe}/R_{we}) \qquad \text{........(5)}$$

where R_{mfe} and R_{we} are equivalent mud filtrate and water resistivities respectively. K is a temperature dependent constant as given below

$$K=65+0.24*T^oC \qquad \text{.......(6)}$$

If R_{mf} at T_f< 0.1 Ω-m, then

$$R_{mfe}= 1.46*R_{mf} \text{ at } T_f +77 \qquad \text{.......(7)}$$

If R_{mf} at T_f> 0.1Ω-m, then

$$R_{mfe}=0.85*R_{mf} \text{ at} T_f. \qquad \text{........(8)}$$

From eq. (5)

$$R_{we}/R_{mfe}= 10^{(SSP/K)} \qquad \text{........(9)}$$

$$R_{we}= R_{mfe}*10^{(SSP/K)} \qquad \text{........(10)}$$

If R_{we}> 0.12 Ω-m, then

$$R_w \text{ (at } T_f) = 0.58-[(6.9 R_{we} + 2.4)] \qquad \text{........(11)}$$

If R_{we}< = 0.12 Ω-m, then

$$R_w \text{ (at } T_f) = (77*R_{we} +5)/ (146-337*R_{we}) \qquad \text{.......(12)}$$

Accordingly R_w is determined. After all the parameters (ϕ, R_w and R_t) have been determined for the hydrocarbon bearing formation, empirical probability distribution function (PDF) and cumulative distribution function (PDF) are created in MS-Excel for all the log derived parameters in order to determine which distribution model should be used for the simulations.

There are two methods to evaluate and analyze the created empirical PDF"s and CDF"s. The first is visual inspection and comparison to the idealized distribution PDF and CDF shapes. The second and more reliable method is to plot the data on probability paper. The NORMSINV function in Excel is used to create a plot that mimics the behavior of the data as if it were plotted on probability paper. The resulting parameter is known as probability transform.For a normal distribution, the data forms a straight line on probability paper and for log-normal distribution, the log of a data forms a straight line on probability paper.

After the distribution model for each parameter is known, 10000 random numbers are generated for simulating probability distribution for Φ, R_w and R_t based on their specific probability distribution models. A different set of random numbers is generated for each parameter. Then S_w is calculated using Archie"s relation (Eq. 1).

The empirical constants a, m and n in eq.(1) and (2) are based on measurements made on cores. In the simulation approach followed here, these empirical 'constants' are held constant and not regarded as random variables although they vary within narrow limits for different tock types, degree of cementation and porosities. For carbonate reservoir, a, m and n are taken constant as 1 ,2 and 2 respectively.

Probability distributions of F, R_w and R_t are used in calculating the same for S_w, For all these parameters, probability distribution function (PDF) and cumulative distribution function (CDF) have been calculated and presented.

Results and Discussion

Figure 1 shows the PDF and CDF generated for Φ, R_w and R_t from the log data. The CDF and PDF plots for Φ and R_w, R_t are characteristic of normally and log-normally distributed random variables respectively.

To cross-check the distribution model, a plot of each log derived parameter vs probability transform is made.

Figure 2 shows that input data for Φ lie in a fairly straight line with some deviation in the tail. It is surmised that porosity is normally distributed with a mean and standard deviation of 0.1722 and 0.0353 respectively.

PDF and CDF plots for R_w and R_t appear similar in character to the log normal distribution. To check if the distribution is truly log-normal, a plot of R_w and R_t versus the probability transform on a semi-log plot is made. For a log-normal distribution, this plot should be a straight line. Both the plots (figure 3 and 4) show a reasonably good straight line. Based on the above analysis, both R_w and R_t are modeled as log-normally distribued with mean and standard deviation of 0.0820 and 0.0038 (for R_w) and 45.57 and 55.28 (for R_t).

A total of 10000 iterations were carried out to create probability distributions (PDF and CDF) for Φ, R_w and R_t with their inferred distribution models. The probability distributions of S_w are obtained by using Archie“s relation (Eq. 1) when PDF“s are substituted for each of the variables (Φ, R_w and R_t).

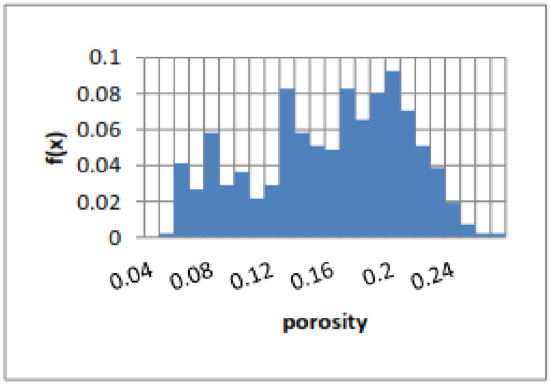

Fig. 1.a PDF for porosity

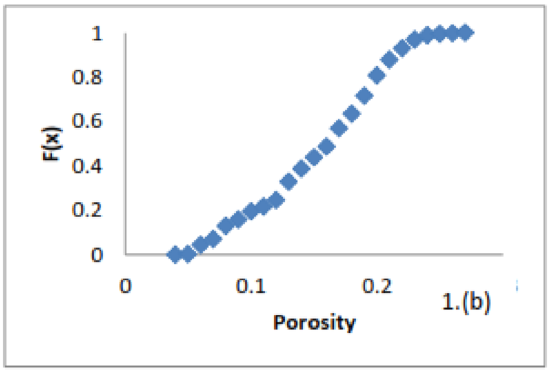

Fig.1.b CDF for porosity

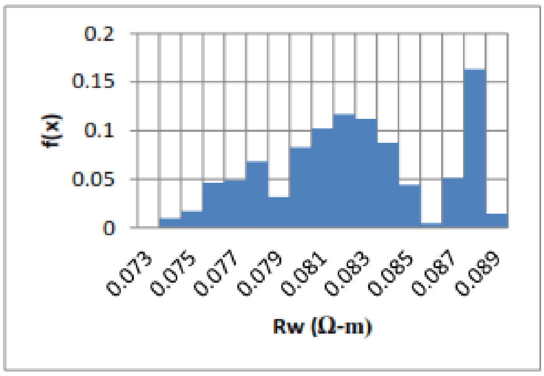

Fig.1.c PDF for R_w

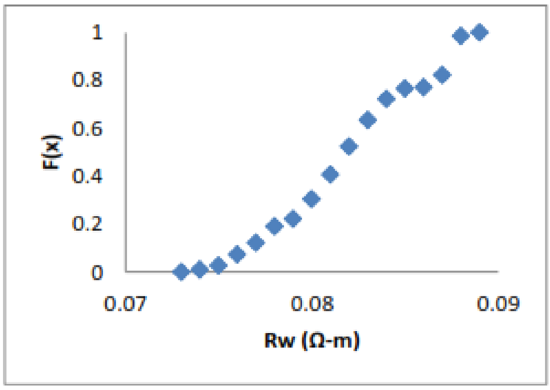

Fig.1.d CDF for R_w

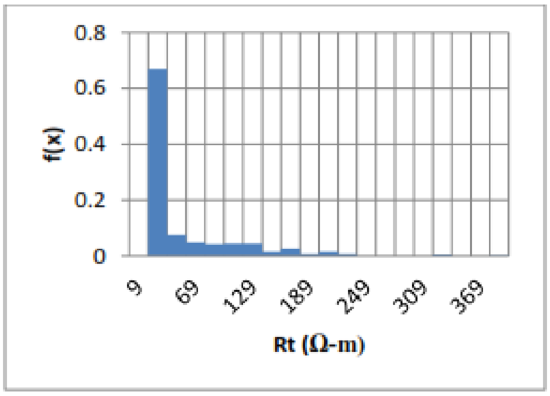

Fig.1.e PDF for R_t

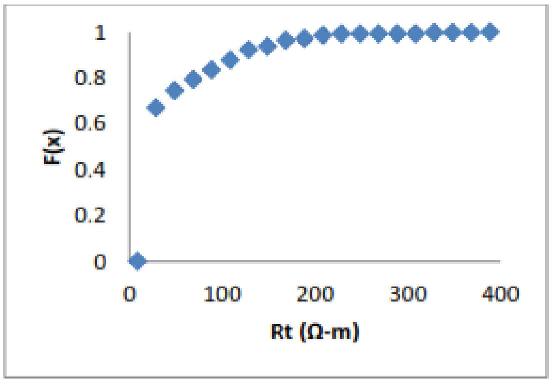

Fig.1.f CDF for R_t

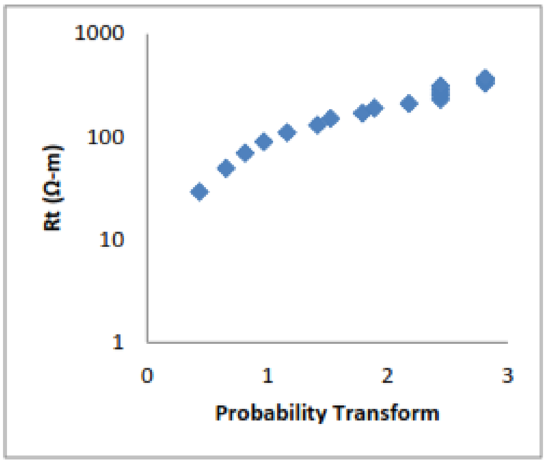

Fig.4. Probability transform plot for R_t

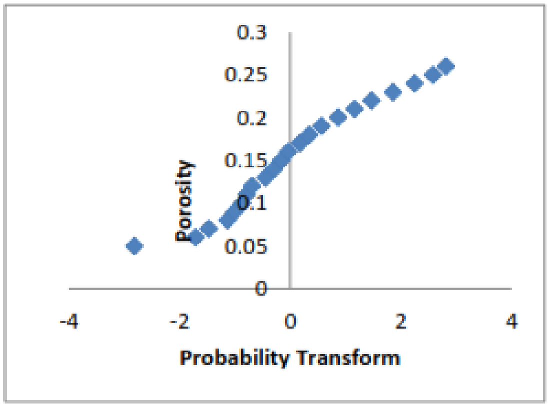

Fig.2. Probability transform plot for porosity

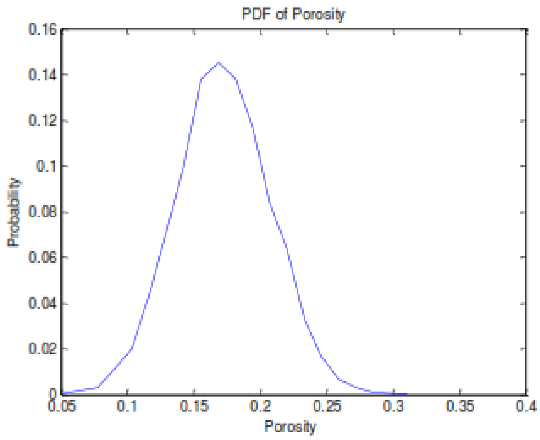

Fig.5. PDF for porosity for 10000 simulations

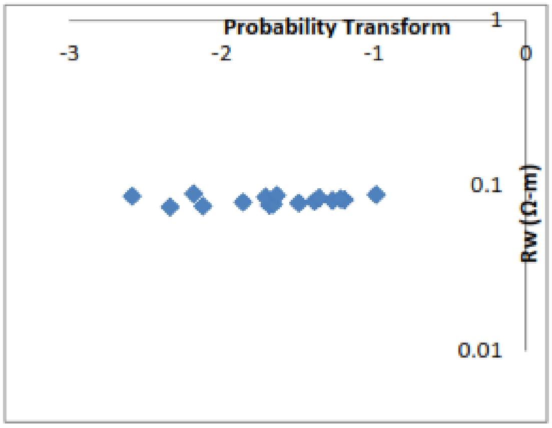

Fig.3. Probability transform plot for R_w

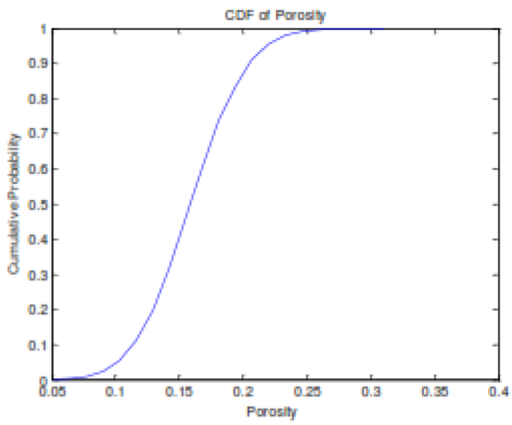

Fig.6. CDF for porosity for 10000 simulations

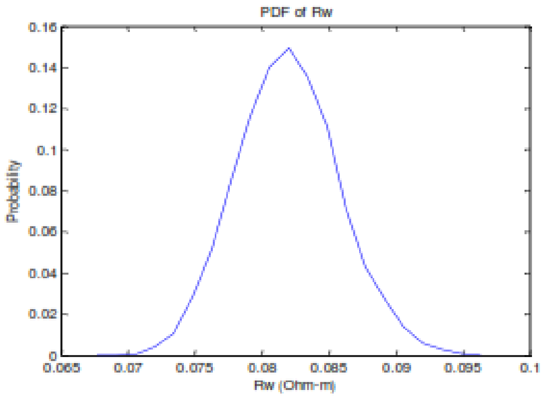

Fig.7. PDF for R_w for 10000 simulations

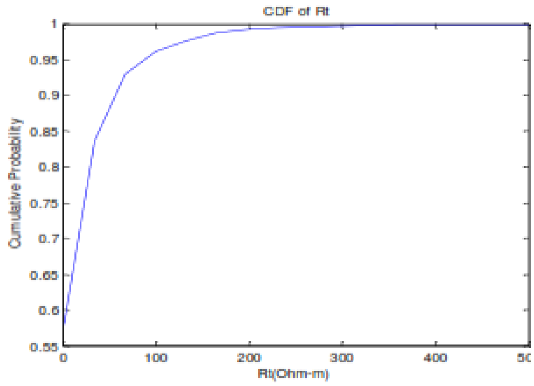

Fig.10. CDF for R_t for 10000 simulations

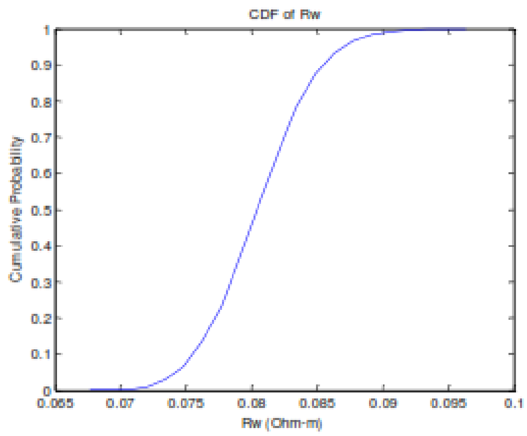

Fig.8. CDF for R_W for 10000 simulations

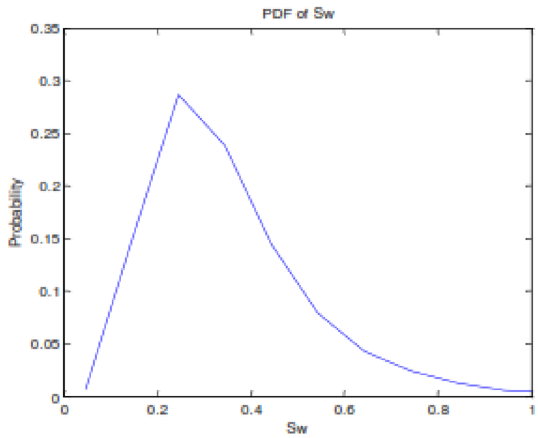

Fig.11. PDF for S_w for 10000 simulations

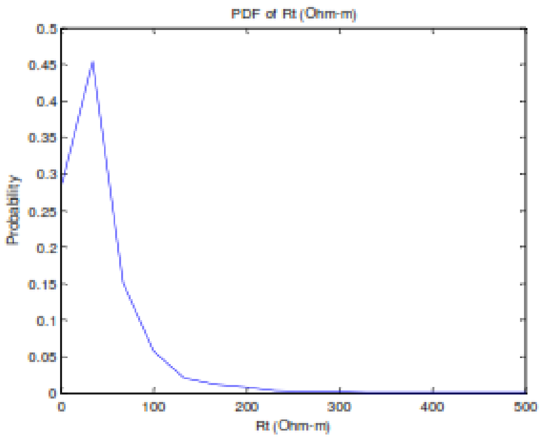

Fig.9. PDF for R_t for 10000 simulations

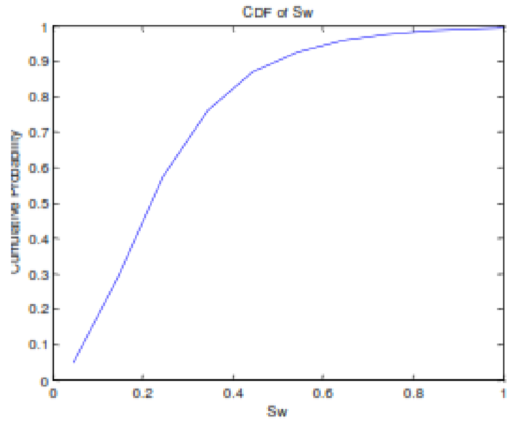

Fig.12. CDF for S_w for 10000 simulations

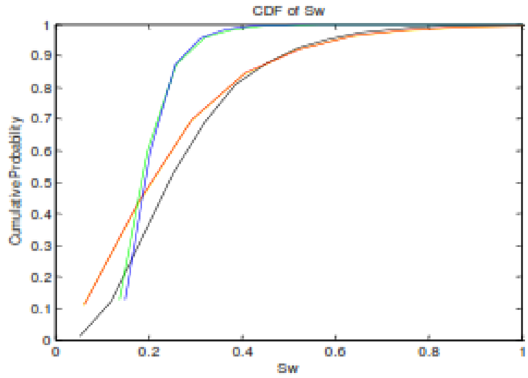

Fig.13. Different curves showing how the variation of each input parameter affects S_w; blue curve: when ϕ varies, while R_w and R_t is constant, green curve: ϕ, R varies while R_t is constant, black curve: when R_w and R varies while ϕ is constant, magenta curve: when R and ϕ varies while R_{wt} is constant, red curve: when all the parameters normally varies. *variation is normally distributed.

Conclusion

I. Monte Carlo simulation not only show what range of value each parameter could have but also show how likely each outcome is.

II. Although distribution for porosity is symmetrical, the distribution for water saturation is not symmetrical because of asymmetrical distributions of R_t and R_w.

III. The nature of the output distribution is Determined by the functional relationship between the input parameters and the type of probability distribution for each of the input parameters.

IV. S_w is more sensitive to the values of Φ compared to variation in R_w and R_t .

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