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On parameter estimation in an inverse problem using optimum and over-parameterized model – a 1D CSEM test case

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Summary

Inversion of Controlled Source Electromagnetic (CSEM) data, for 1D model has been attempted as an optimum and over-parameterized inverse problem. For these tests two scenarios has been considered. First scenario, when the number of layers in true model is available and second scenario is when no such information is available. For first scenario attempt is made to solve problem with optimum parameterization while for second scenario problem is solved using over-parameterization. To discuss these two scenarios we have utilized two schemes of inversion, first (scheme 1) only invert for conductivity and second (scheme 2) which invert for both conductivity and thickness of layers. In case of first scenario, inversion scheme 1 needs a systematic trial and error approach to guess the thickness and depth of resistive layers. In inversion scheme 2 inverted model parameter can be estimated in one pass. For second scenario, due to over-parameterization, problem is solved for the smooth (minimum information) solution. Due to over-parameterization parameter (conductivity) estimation developed mild oscillation around sharp conductivity contrast and in deeper parts of model space. These observations are valid for both the schemes of inversion. It has also been noted that inversion scheme 2 required more educated guess than scheme 1.

***Keywords**: Controlled source electromagnetic (CSEM,) Inversion, Over-parameterization*

Introduction

The marine Controlled Source Electromagnetic (CSEM) technique utilizes the low frequency Electromagnetic (EM) signals which are recorded at far away receivers placed at the sea floor (Chave and Cox, 1982). In most of the acquisitions the source is towed at couple of decade meters above sea floor. Due to the availability of advanced receivers, signal can be recorded at tens of thousands of meters away from the source with acceptable noise level. In the presence of resistive layers, EM energy travels as guided wave through them with much less attenuation. Field trials have shown the ability of this method to detect a thin resistive layer in deep water environment (Ellingsrud et al. 2002). This has led to huge commercial interest in the development of the method (Constable and Srnka, 2007).

Due to the air wave interference in shallow water it faces some challenges. Many workers have studied CSEM responses in shallow water environment and have made significant improvement in this direction. CSEM technique has also seen active research interest in 2D and 3D numerical. Instrumentation in this field has also witnessed huge development. It is perhaps one of the requirements for any geophysical method, particularly those that have to operate in tough environment conditions. These days a range of CSEM receivers are available and some of these can record all three components of electric and magnetic field. Inline towed source and horizontal receiver recording is most popular. 1D CSEM inversion of synthetic data study has shown that horizontal component of electric (magnetic) field recorded with inline receiver configuration for horizontally towed electric dipole source alone can resolve the thin resistive layers (Key, 2009).

This study is aimed to discuss the effect of optimum and over parameterization of inverse problem in the context of 1D CSEM data. Motivation for this study comes from the fact that most of the inverse problems are over parameterized particularly when prior information about geometry of true model is not available. In case of vertical electrical sounding data Gupta et.al (1997) have

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noted that over-parameterization leads to spurious oscillation in the inverted resistivity values, centered around true resistivity values. In this study the effect of optimum and over parameterization is presented using two methods of 1D CSEM inversion for isotropic layered Earth model. First is to invert for conductivity with fixed layer thicknesses (here after called as scheme 1) and second is to invert for both conductivity and thickness of layers (here after called as scheme 2).

Method

This section briefly presents forward and inverse modelling method used in this study. EM response due to a dipole embedded in layered media is very well studied by many researchers. For the forward modeling we have utilized the method presented by Loseth and Ursin (2007). They have presented CSEM modelling for general anisotropic layers media. However, here we have presented results only for isotropic layered model.

In layered model, Maxwell's equation can be expressed in frequency wave number domain. This formulation leads to a differential equation which can be represented by a field vector, a system matrix and a vector with source information. Element of system matrix depend on material properties and horizontal slowness of a particular homogeneous layer under consideration. System matrix can be diagonalized using eigen-mode analysis technique and this transforms field vector into mode field vector. Now these decoupled differential equations can be solved for any homogeneous layer. Using the boundary conditions, eigenmode fields can also be computed across the interface. This formulation leads to a recursive scheme for computing the response due to stack of layers. In case of isotropic media it turns out to be a scalar algebra. Now using the eigen vectors of system matrix, mode fields vector can be transform into field vector. Further a two dimension Fourier transform can be performed and field response can be obtained in frequency domain. Due to horizontal symmetry 2D Fourier transform can be replaced by Hankel transform and it can be efficiently computed using digital filter method (Ghosh, 1971; Anderson, 1983). By this method one can compute the EM response at any point in the media using the known source for layered Earth model.

Inversion of CSEM data is a nonlinear problem and one of the ways to solve it is quasi linearization approach. In this scheme correction in the model parameter is computed from the residual between observed and predicted responses due to current updated model. Hence an initial guess model is required to start with this scheme. A kernel matrix which maps the residual vector into the parameter correction vector is a kind of generalized inverse of field sensitivity (Jacobian) matrix. Elements of Jacobian matrix are defined as derivatives of field response with respect to each model parameter for all source-receiver combination. We have utilized the forward difference formula to compute these derivatives numerically. For each sourcereceiver combination one has to solve np+1 forward problems, where np is the number of model parameters to be updated and hence the computation of Jacobian is computationally expensive. This demands an efficient forward modeling algorithm. It is also vital to minimize the number of model parameters using some prior information, if available. Another challenge in EM inversion is ill-posedness of the inverse problem. This is resolved by regularizing the inverse problem and in this study Levenberg-Marquardt algorithm (damped least square) has been implemented. Hence the regularized inverse problem takes the following from,

$$\delta \mathrm{P} = (\mathbf{J}^{\mathbf{T}}\mathbf{J} + \mu\mathbf{I})^{-1}\mathbf{J}^{\mathbf{T}}\delta \mathrm{R}\,, \tag{1}$$

Where δP is correction in model parameter, $\mathbf{J}$ ($\mathbf{J}^{\mathrm{T}} =$ transpose of $\mathbf{J}$) is Jacobian matrix, μ is Marquardt parameter and δR is residual.

Equation (1) is solved to obtain the correction for model parameters. Solution of inverse problem improves with each iteration and the process is stopped when desired/accuracy is achieved. In EM inversion, conductivities of the layers (scheme 1) alone or conductivities and thickness both can be taken as model parameters (scheme 2).

Tests

Here we will discuss synthetic inversion test results of the two scenarios. First is the case when number of layers in the true model is known and second is when no such information is available. For the first scenario an attempt is made to solve the problem using optimum-parameterization while for second scenario problem is solved using overparameterization. For this study, we have developed a FORTRAN code for forward and inverse modeling based on the algorithm explained in last section. However, the synthetic observed data has been generated using WHAM: web hosted active source modelling code which is web interface of 1DDIPOLE code of Key (2009). Data has been generated for two frequencies (1.0Hz and 0.25Hz) and 2% Gaussian random noise was added to the data. The true model for these tests is same as taken by Key (2009). This model comprises five layers. From top to bottom as air layer of conductivity of 10^{-12} S/m, sea water layer of conductivity 3.3 S/m and thickness 1000m, sediment layer of conductivity 1 S/m and thickness 1000m, reservoir layer of conductivity 0.01 S/m and thickness 100m, sediment layer of conductivity 1 S/m respectively. We have also considered that thickness and conductivity of sea water layer is known as prior information. Hence model parameter space for the inversion is below sea bottom.

Scenario 1: when number of layers is known

In inversion scheme 1 we can only invert for conductivity, hence the geometry of layers can only be estimated by trial and error method. Here, we have used a systematic approach to guess the geometry of the layers. First thickness of the resistive layer is fixed by some assumed guess value and then the depth of resistor is increase by an increment of same amount as the assumed thickness of resistive layer. Now the model with least RMS error is selected for next stage where this resistive layer is further divide into two half. Again model with least RMS is considered for next step and so on. This process is stopped when the RMS error reach to acceptable limit. Here, we started with 500m layer thickness and 12 trials were needed to get the inverted model with acceptable RMS error. Inversion scheme 2 is also tested for this scenario and there is only one pass of inversion is required for this scheme. It has been noted during the tests that even though scheme 2 required only one pass of inversion but it required more educated initial guess of conductivity than scheme 1. It has been observed that scheme 2 required an initial guess of conductivity approximately 20% inside the interval of 1 S/m - 0.1 S/m, which is the limit of conductivity value of true model in the inverted model space. here we have worked with half space of conductivity 0.1 S/m below sea bottom as initial guess. Scheme 1 was able to invert the model with all such initial guesses, where conductivity of the half space is taken of the order of most marine sediment types found in nature. This is due to the fact that nonlinearity in layers geometry parameters is more than its conductivity. Inverted model parameters and RMS error plot of these test is illustrated in figure 1. One point should be noted here that for multilayers true model these trial and error methods may become cumbersome. Even this strategy of identifying layers geometry may not be practically viable if the number of layers is large. On the other hand scheme 2 demands even better educated initial guess for multilayer true model and initial guess as half space below sea bottom may not succeed.

Scenario 2: when number of layers is not known

In this scenario, problem is posed as over-parameterized inverse problem. Inversion using scheme 1 required model space to be discretized in to large number of thin layers of equal thicknesses. Here three tests are presented with layer thicknesses of 50m, 100m, and 200m, constituting 80 layers, 40 layers and 20 layers parameter space respectively. It has been noted that inverted parameters are mildly oscillatory due to over-parameterization particularly around the resistor and in the deeper part of the part of the model space. Oscillatory behavior of conductivity is enhanced as the number of layers (parameters) is increased. Inverted model with 80 layers has developed some artifacts in the shallow part also. These signatures are complementary with observation made by others studies (Gupta et. al. 1997) for over-parameterized invers problems. Because thickness of resistive layer is less than layers thickness in case of 20 layers case hence conductivity estimation of resistive layer in this case is not as good as of others two. Results of these tests are illustrated in figure 2. To test scheme 2 for overparameterized invers problem, three tests are performe where we seeks 5, 8 and 15 layers model. Inversion results of scheme 2 are also in agreement with observation made for scheme 1 as far as oscillatory behavior of conductivity of inverted model is concern. But it is manly present in the deeper part of model space (below resistive layer) where sensitivity is comparatively less. Inverted model parameters and RMS error plot of these test is illustrated in figure 3.

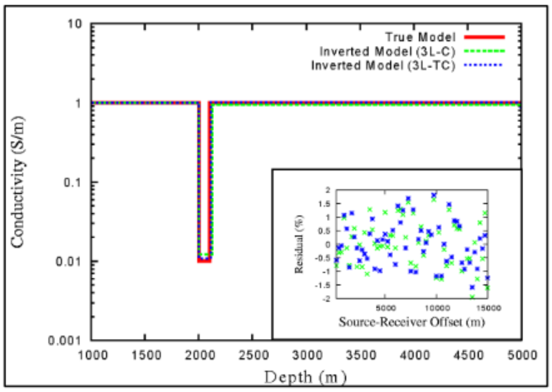

Figure1. True model, inverted model (optimum parameterization) and RMS misfit plot (inset). (3L-C) 3 layers inverted model using scheme 1 and (3L-TC) three layers inverted model using scheme 2.

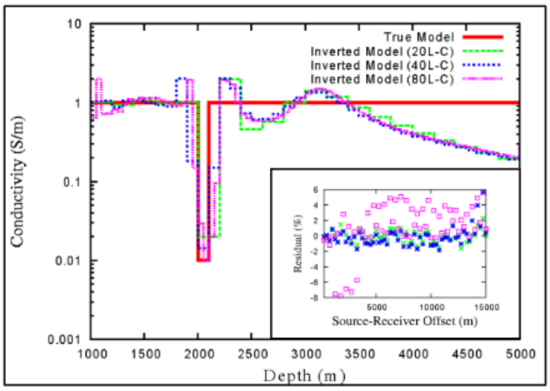

Figure2. True model, inverted model (over-parameterization) and RMS misfit plot (inset). (20L-C) 20 layers inverted model, (40L-C) 40 layers inverted model, (80L-C) 80 layers inverted model using scheme 1.

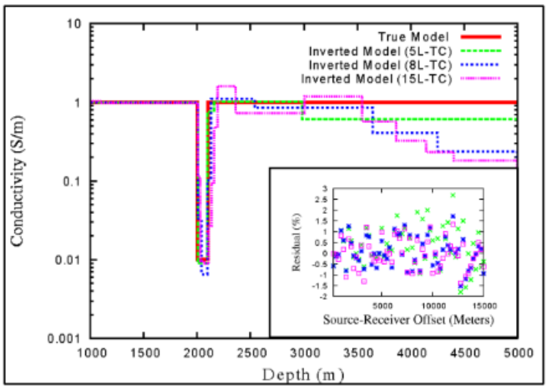

Figure3. True model, inverted model (over-parameterization) and RMS misfit plot (inset). (5L-TC) 20 layers inverted model, (8L-C) 8 layers inverted model, (15L-C) 15 layers inverted model using scheme 2.

Conclusion

To discuss the optimum and over-parameterized inverse problem for 1D CSEM data we have considered two scenarios. First scenario is, when information about the number of layers in true model is available and second scenario is when no such information is available. Solution of the inverse problem for first scenario is attempted by optimum-parameterization of model space while for second scenario problem is solved using over-parameterization of model space. For both the scenario tests are done with two schemes of inversion. First scheme (scheme 1) inverts for only conductivity as model parameter considering fixed layer thicknesses and second scheme (scheme 2) inverts for both conductivity and thicknesses of the layers. For first scenario inversion scheme 1 required a systematic trial and error approach for optimum parameterization of model space while inversion scheme 2 can achieve the solution in one go. For second scenario, it has been noted that due to over-parameterization, inverted model becomes mildly oscillatory around resistive body and also in the deeper part of the model space. This behavior is consistent in both the inversion schemes but in scheme 2 it is mainly in the deeper part of the model space while in scheme 1 further increase in number of parameters leads to some artifacts in shallow part of the inverted model also. It has also been noted that inversion scheme 2 required more educated initial guess of conductivity values than the scheme 1. This is due to the higher nonlinearly in geometry parameter then in conductivity.

This study is done on simple 1D model. Hence these comments cannot be taken in general and need to be validated for multilayer or 2D/3D true model.

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WHAM: web hosted active source modelling (http://marineemlab.ucsd.edu/wham/wham_form.html)