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Bezier curves and representation of reservoir extent maps

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Summary

The final products/maps of any synergistic geo-scientific interpretation project of an E&P company are stored as CGM (Computer Graphics Meta) files. These include (1) time and depth maps at various levels and (2) maps of sand extent and thickness in established or development fields used for reserve estimates. Normally, CGM files for maps of former type are generated in various interpretation workstations whereas files of the latter type are produced by computer graphics software packages like CorelDraw. However, the nature of representation of different cultural and geological features in these output CGM files varies widely. In particular, it has been found that the sand extent and contour maps are not represented by poly-line elements or polygons but a series of "control points" out of which the actual shapes are to be reconstructed as ***Bezier Curves****. Deciphering and reconstruction of Bezier curves are keys to creating a corporate database of reserve estimate maps.*

Keywords*: Bezier Curves, CGM files*

Introduction

For the purpose of creating a comprehensive database of all G&G and reserve estimate maps of an E&P company, a user-friendly methodology has been developed to decode CGM plot files and convert them into ASCII files so that after geo-referencing, all the features contained in them can be imported into GIS (Geographical Information System) database. In doing so, it has been seen that most features like international/state/block boundaries, contours, faults, survey lines are stored as line elements or polygons. The exception, however, was observed while deciphering the sand extent/thickness features in reserve estimate maps prepared in CorelDraw and then exported as CGM files. It was found from the encoding specifications that they are stored as control points of a set of smoothly joined Bezier curves whose undulations or curvatures (representing the actual shapes of the sand maps) are governed by the relative location of these "control points". This necessitated the study of Bezier curves in mathematical literature which resulted in the incorporation of algorithm for constructing Bezier curves from given control points in the CGM file decoding software.

Methodology – What are Bezier curves?

The mathematical basis for Bézier curves - the Bernstein polynomial - has been known since 1912, but its applicability to graphics was understood half a century later. Bézier curves were widely publicized in 1962 by the French engineer Pierre Bézier, who used them to design automobile bodies at Renault.

A Bézier curve is defined by a set of *control points* $\mathbf{P}_0$ through $\mathbf{P}_n$, where n is called its order (n =1 for linear, 2 for quadratic, etc.). The first and last control points are always the end points of the curve; however, the intermediate control points (if any) generally do not lie on the curve.

Linear Bézier curves

Given points $\mathbf{P}_0$ and $\mathbf{P}_1$, a linear Bézier curve is simply a straight line between those two points. The curve is given by

$$\mathbf{B}(t) = \mathbf{P_0} + t(\mathbf{P_1} - \mathbf{P_0}) = (1-t)\mathbf{P_0} + t\mathbf{P_1}\,, t \in [0,1]$$

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and is equivalent to linear interpolation.

Quadratic Bézier curves

A quadratic Bézier curve is the path traced by the function $\mathbf{B}(t)$, given points $\mathbf{P}_0$, $\mathbf{P}_1$, and $\mathbf{P}_2$,

$$\mathbf{B}(t) = (1-t)[(1-t)\mathbf{P_0} + t\mathbf{P_1}] + t[(1-t)\mathbf{P_1} + t\mathbf{P_2}]$$

which can be interpreted as the linear interpolant of corresponding points on the linear Bézier curves from $\mathbf{P}_0$ to $\mathbf{P}_1$ and from $\mathbf{P}_1$ to $\mathbf{P}_2$ respectively. Rearranging the preceding equation yields:

$$\mathbf{B}(t) = (1-t)^2\mathbf{P_0} + 2(1-t)t\mathbf{P_1} + t^2\mathbf{P_2}\,, \quad t \in [0,1]$$

The derivative of the Bézier curve with respect to t is

$$\mathbf{B}'(t) = 2(1-t)(\mathbf{P_1} - \mathbf{P_0}) + 2t(\mathbf{P_2} - \mathbf{P_1})\,, t \in [0,1]$$

from which it can be concluded that the tangents to the curve at $\mathbf{P}_0$ and $\mathbf{P}_2$ intersect at $\mathbf{P}_1$. As t increases from 0 to 1, the curve departs from $\mathbf{P}_0$ in the direction of $\mathbf{P}_1$, then bends to arrive at $\mathbf{P}_2$ in the direction from $\mathbf{P}_1$.

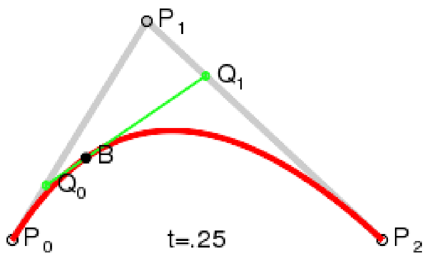


Fig. 1: Quadratic Bezier curve with three control points P_0 , P_1 and P_2

Cubic Bézier curves

Four points $\mathbf{P}_0$, $\mathbf{P}_1$, $\mathbf{P}_2$ and $\mathbf{P}_3$ in the plane or in higher-dimensional space define a cubic Bézier curve. The curve starts at $\mathbf{P}_0$ going toward $\mathbf{P}_1$ and arrives at $\mathbf{P}_3$ coming from the direction of $\mathbf{P}_2$. Usually, it will not pass through $\mathbf{P}_1$ or $\mathbf{P}_2$; these points are only there to provide directional information. The distance between $\mathbf{P}_0$ and $\mathbf{P}_1$ determines "how long" the curve moves into direction $\mathbf{P}_2$ before turning towards $\mathbf{P}_3$.

Writing $\mathbf{B}_{\mathbf{P}i,\mathbf{P}j,\mathbf{P}k}(t)$ for the quadratic Bézier curve defined by points $\mathbf{P}_i$, $\mathbf{P}_j$, and $\mathbf{P}_k$, the cubic Bézier curve can be defined as a linear combination of two quadratic Bézier curves:

$$\mathbf{B}(t) = (1-t)\mathbf{B}_{\mathbf{P_0},\mathbf{P_1},\mathbf{P_2}} + t\mathbf{B}_{\mathbf{P_1},\mathbf{P_2},\mathbf{P_3}}\,, t \in [0,1].$$

The explicit form of the curve is:

$$\mathbf{B}(t) = (1-t)^3\mathbf{P_0} + 3(1-t)^2t\mathbf{P_1} + 3(1-t)t^2\mathbf{P_2} + t^3\mathbf{P_3}$$

For some choices of $\mathbf{P}_1$ and $\mathbf{P}_2$ the curve may intersect itself, or contain a cusp.

Any series of any 4 distinct points can be converted to a cubic Bézier curve that goes through all 4 points in order. Given the starting and ending point of some cubic Bézier curve, and any two other distinct points along that curve, the control points for the original Bézier curve can be recovered.

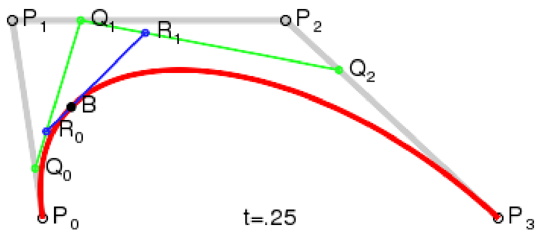


Fig. 2: Cubic Bezier curve with four control points P_0, P_1, P_2 and P_3

Application to sand extent/thickness in reserve estimate maps

The CGM files generated by graphics software like CorelDraw stores the control points as succession of a set of 3 or 4 points. It turns out that the actual shape of the thickness contours is properly reproduced by a cubic Bezier interpolation curve where the last control point of a segment of 4 points is the first one for the next segment. The coordinates of these points as decoded from the CGM files, however, are some arbitrary numbers (henceforth referred to as “device coordinates”) that in no way resemble the “user coordinates” like the easting and northing. They, therefore, need to be geo-referenced to the actual map coordinates (defined according to a particular projection system) by a one-to-one mapping from the “device plane” to the “user plane”.

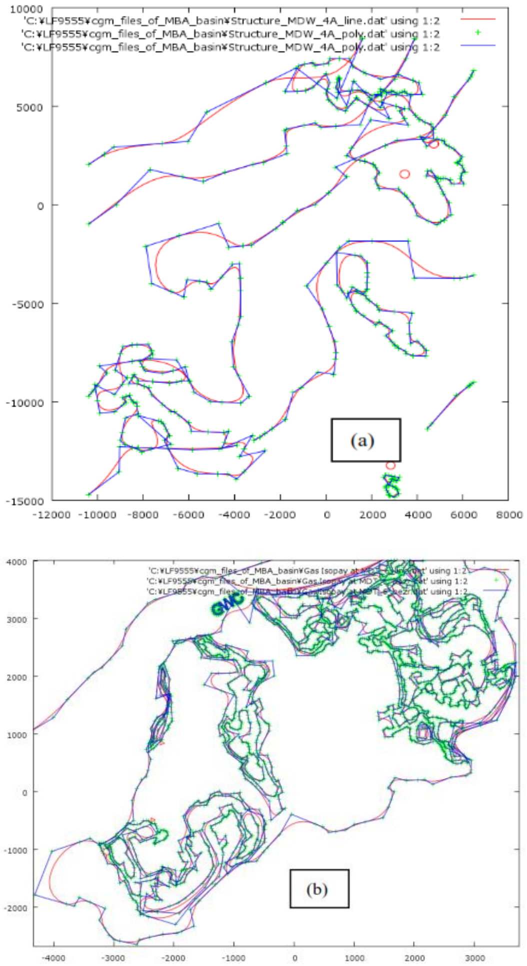


Fig. 3: Gas isopay map contours as Bezier curves (a), (b) (in red) along with the control points (joined in blue/green). Note that the graduations along the borders are in arbitrary device coordinates.

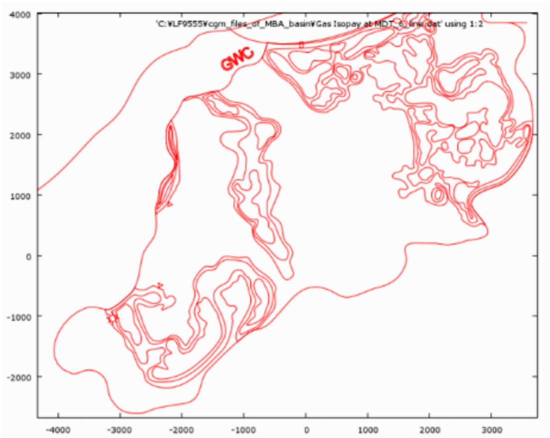


Fig. 4: Gas isopay map contours as Bezier curves without the control points. The graduations along the borders are in device coordinates.

Mapping between device and user coordinate planes

After accomplishing the task of decoding device coordinates from the CGM file, a fairly accurate mapping of user coordinates (easting and northing) on to these device coordinates is very important for the purpose of deriving the actual geographical locations. This is done by establishing the correspondence of the lower left and the upper right corners on the device coordinate plane and the user coordinate plane. While the device coordinates are found out from the extracted data, the user coordinates are carefully read out from the G&G or reserve estimate map. It is to be ensured that the ratios of height to width in both coordinates systems are same up to some desired accuracy for isotropic mapping between them, that is (see Fig. 5),

$$(yu2-yu1)/(xu2-xu1)\approx(yd2-yd1)/(xd2-xd1)$$

where (*xd1*, *yd1*) and (*xd2*, *yd2*) are the device coordinates of the lower left and upper right corners of a map and (*xu1*, *yu1*) and (*xu2*, *yu2*) are the corresponding user coordinates. Matching of aspect ratios to two places of decimal is considered to be accurate in case of G&G maps created in workstations where the map boundary is graduated in grid coordinates. In reserve estimate maps, particularly in old legacy ones, where boundary graduations are in latitude and longitude and the drawings are generally not so accurate, matching to one place of decimal may be considered as satisfactory. This constitutes a stringent quality check before proceeding further.

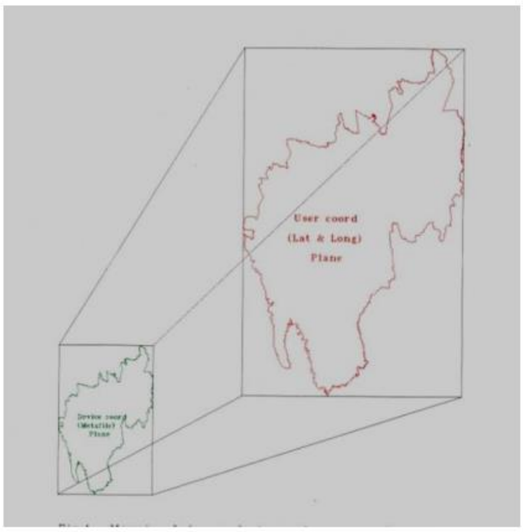


Fig. 5: Mapping between device (green) and user (red) coordinates

Segregation of different layers of a map

After conversion to the "user coordinates", the data file is ready to be imported into the GIS (in particular, the ArcGIS whose displays are shown below). Initially, all the data is displayed as discrete points of the same color as a single or composite layer. To separate different layers or features like block boundaries, seismic lines, contours, faults etc., one uses attributes such as (1) line type, line width and line color for open curves (poly-line/Bezier) or (2) edge type, edge color and fill color for closed curves(polygon/Bezier). These attributes are also extracted from the CGM file.

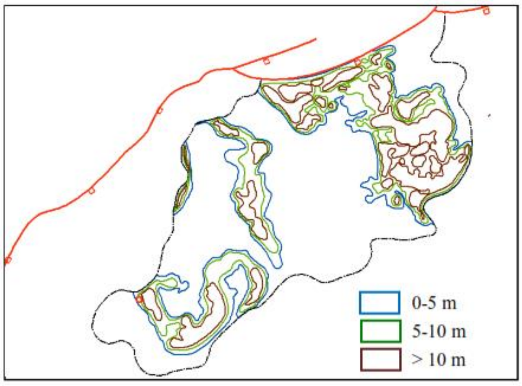


Fig. 6: The isopay map shown in Fig. 4 after import to ArcGIS. Pay ticknesses are shown in different colors.

Using the query language in the "select by attribute" dialog box of ArcGIS, various pay thickness contours, gas water contact boundary, fault etc. are marked with different colours. A zoomed lower left part of the thickness map is shown in Fig. 7. The cubic Bezier curve interpolation, with four control points, in the left is clear with the fourth point of one segment (points 1 to 4) being the first point of the next (points 4 to 7). The two segments are joined smoothly at point number 4.

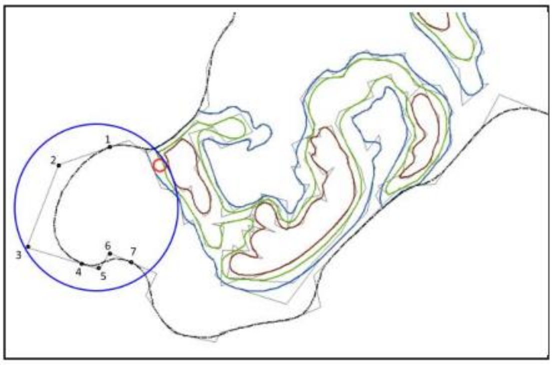


Fig. 7: Points marked as (1, 2, 3, 4) and (4, 5, 6, 7) are control points for two adjacent cubic Bezier curves joined smoothly at point 4

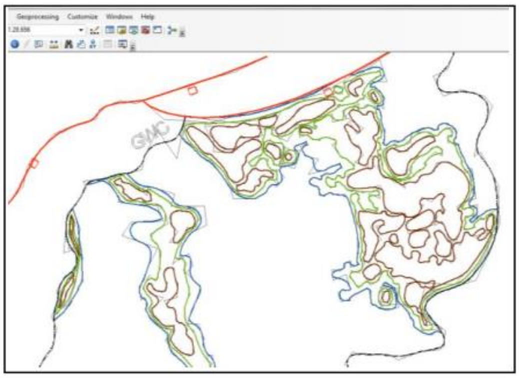


Fig. 8: Similar Bezier interpolation in the upper middle part of the reservoir map

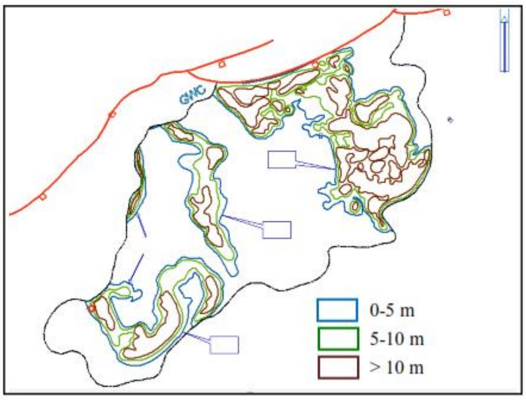


Fig. 9: Final isopay map as smooth Bezier curves shown without the control points. Different edge colors (blue, green, brown) indicate different pay thicknesses. The bounding fault is shown in red.

Salient features of the Software

Representative flow-chart for the software is given in Fig. 10. The software, written in FORTRAN, can be installed on UNIX based system as well as on PC. The name of the main CGM conversion module is **"decode_cgmfile"** and the subsequent module for transformation of arbitrary "device" coordinates to the "user" coordinates is called **"dev_to_user".**

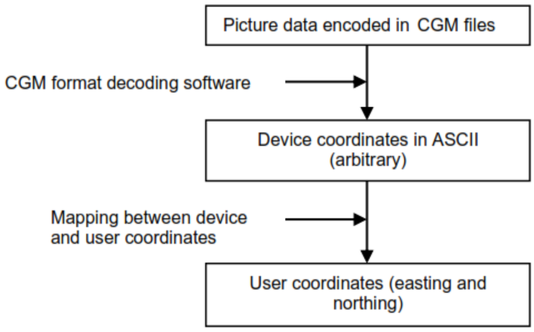
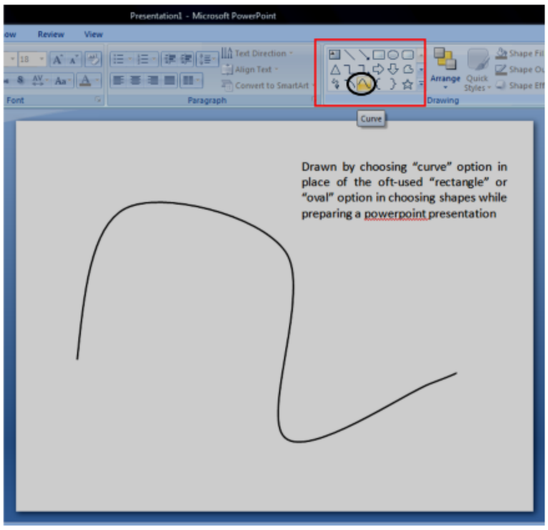


Fig. 10: Software flowchart

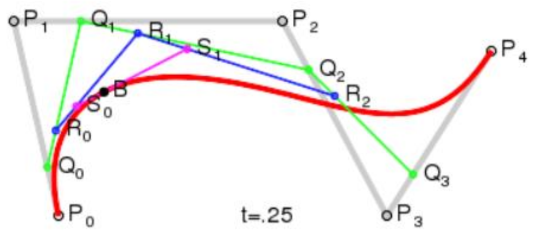
From the study of a number of CGM files from different sources, it has been found that the device coordinates could be encoded in either of 2, 3 or 4 bytes and as any of integers, fixed point real or floating point real numbers. The module **decode_cgmfile** automatically detects this and proceeds to convert the contents of the CGM file so that the process is transparent to the user.

Conclusion and Remarks

In attempting to create a database of reservoir extent maps drawn in graphics software like CorelDraw and exported as CGM files, it was found that they are encoded as a set of control points governing the shape of the contours. These shapes are called Bezier curves which are reconstructed by giving weights to the relative positions of the control points as discussed in Section II. These are very similar to the drawings that are generated by choosing the “curve” option from available shapes in preparing a powerpoint presentation.



Compare with the quartic(fourth order) Bézier curve below.



The procedure to digitize reserve estimate maps also include G&G interpretation maps at different depth and time levels preserved as CGM files and store them in a layered GIS-based database structure facilitating corporate decision making.

Finally, it may be mentioned that the real utility of creating a database is when any required information can retrieved with ease. In the present case, all the archived maps stored in ArcGIS as .shp files, can be quickly imported back and superposed on any other maps on any of the new generation interpretation workstation like R5000, Petrel, Kingdom, whenever necessary. This will enable the users to re-create the map features with minimal effort.

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