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Low Frequency Seismic Reflection Coefficients at Non-Welded Interfaces

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Summary

A common assumption in reflection seismology is that the interfaces are perfectly welded, implying continuity of both traction and particle velocity. At an imperfectly coupled (or non-welded) interface, like a fault or a fracture, traction can generally still be assumed continuous, but particle velocity can not. Such interfaces can be described with a linear slip boundary condition, where the particle velocity discontinuity is assumed to relate linearly to the interface traction through the interface compliance. We evaluate seismic reflections at non-welded interfaces sandwiched between general anisotropic media and show that at low frequencies the real parts of the reflection coefficients can be approximated by the responses of equivalent welded interfaces, whereas the imaginary parts can be related directly to the interface compliances.

Introduction

Faults or fractures can cause a discontinuity in the particle velocity of an elastic wave field. In linear slip theory, we assume that the particle velocity discontinuity can be related linearly to the interface traction through the normal and tangential interface compliances, assuming continuity of traction (Schoenberg, 1980). Seismic reflection coefficients at a single non-welded interface have been studied by various authors analytically, numerically and in laboratory experiments (Pyrak-Nolte, 1990, Haugen and Schoenberg, 2000). Nakagawa et al. (2000) showed that fractures under shear stress may give wave conversion at normal incidence, which can be explained by extending the linear slip model with coupling compliances. Reflection and transmission coefficients at non-welded interfaces are complex-valued, yielding distortions of both amplitude and phase of a reflecting or transmitting wave field. We take a similar approach as Haugen and Schoenberg (2000) by modeling seismic reflection coefficients at a non-welded interface between two general anisotropic layers. We show that at low frequencies the real parts of the reflection coefficients approximate the coefficients of an equivalent welded interface and the imaginary parts can be related directly to the interface compliances.

Linear Slip Theory

We start with a brief review of linear slip theory and the derivation of seismic reflection coefficients at a non-welded interface between two general anisotropic layers. Consider a horizontal non-welded interface in 3-dimensional space. We apply temporal and spatial Fourier transformation with respect to time and the

lateral coordinates, respectively. We assume that the traction vector $\mathbf{t}_3$ is continuous at the interface,

whereas the particle velocity vector $\mathbf{v}$ is not. The boundary condition for linear slip can now be formulated as

$$\begin{pmatrix} \mathbf{v}_{II} \\ \boldsymbol{\tau}_{3,II} \end{pmatrix} = \begin{pmatrix} \mathbf{I} & i\omega\mathbf{Z} \\ \mathbf{O} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{v}_{I} \\ \boldsymbol{\tau}_{3,I} \end{pmatrix}. \quad (1)$$

Here I and II denote the upper and lower media, respectively, i denotes the imaginary unit and ω is

$$\mathbf{Z} = \begin{pmatrix} Z_T & Z_{12} & Z_{13} \\ Z_{12} & Z_T & Z_{23} \\ Z_{13} & Z_{23} & Z_N \end{pmatrix}, \quad (2)$$

where Z_N is the normal compliance, Z_T is the tangential compliance and Z_{12}, Z_{13} and Z_{23} are coupling compliances. All interface compliances are assumed real-valued, implying that no energy is dissipated at the interface.

Wave field quantities $\mathbf{v}$ and $\boldsymbol{\tau}_3$ can be decomposed into $\mathbf{w}^+$ and $\mathbf{w}^-$, being the wave vectors of the downgoing and upgoing wave fields, respectively (Ursin, 1983; Wapenaar and Berkhout, 1989). The wave vectors are related to the particle velocity and traction through

$$\begin{pmatrix} \mathbf{v} \\ \boldsymbol{\tau}_3 \end{pmatrix} = \begin{pmatrix} \mathbf{L}_1^+ & \mathbf{L}_1^- \\ \mathbf{L}_2^+ & \mathbf{L}_2^- \end{pmatrix} \begin{pmatrix} \mathbf{w}^+ \\ \mathbf{w}^- \end{pmatrix}, \tag{3}$$

where $\mathbf{L}_1^\pm$ and $\mathbf{L}_2^\pm$ are composition matrices that can be computed numerically. Consider an incident downgoing wave field in the upper medium $\mathbf{w}_I^+$, reflecting at the interface and $\mathbf{w}_{II}^-$ being absent. We introduce reflection matrix $\mathbf{R}$ to describe the upgoing wave field in the upper medium as $\mathbf{w}_I^- = \mathbf{R}\mathbf{w}_I^+$. Similarly, a 3x3 transmission matrix $\mathbf{T}$ is introduced via $\mathbf{w}_{II}^+ = \mathbf{T}\mathbf{w}_I^+$. Substituting these formulations into boundary condition (2) yields

$$\begin{pmatrix} \mathbf{L}_{1,II}^+ & \mathbf{L}_{1,II}^- \\ \mathbf{L}_{2,II}^+ & \mathbf{L}_{2,II}^- \end{pmatrix} \begin{pmatrix} \mathbf{T} \\ \mathbf{O} \end{pmatrix} = \begin{pmatrix} \mathbf{I} & i\omega\mathbf{Z} \\ \mathbf{O} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{L}_{1,I}^+ & \mathbf{L}_{1,I}^- \\ \mathbf{L}_{2,I}^+ & \mathbf{L}_{2,I}^- \end{pmatrix} \begin{pmatrix} \mathbf{I} \\ \mathbf{R} \end{pmatrix}. \tag{4}$$

We can solve this equation for $\mathbf{R}$ as

$$\mathbf{R} = -\left(\boldsymbol{\Xi}^- - i\omega\mathbf{Z}\mathbf{L}_{2,I}^-\right)^{-1}\left(\boldsymbol{\Xi}^+ - i\omega\mathbf{Z}\mathbf{L}_{2,I}^+\right), \tag{5}$$

with

$$\boldsymbol{\Xi}^\pm = \mathbf{L}_{1,II}^+\left(\mathbf{L}_{2,II}^+\right)^{-1}\mathbf{L}_{2,I}^\pm - \mathbf{L}_{1,I}^\pm . \tag{6}$$

Equation (5) allows us to compute exact reflection coefficients of all (quasi) wave modes and their conversions at a non-welded interface between general anisotropic media and is comparable with equation (A-10) of Haugen and Schoenberg (2000).

LOW FREQUENCY APPROXIMATION

At low frequencies the elements of $\omega\mathbf{Z}\mathbf{L}_{2,I}^-$ are small compared to their counterparts of $\boldsymbol{\Xi}^-$ and equation (5) can be split in a real part that resembles the reflection matrix of an equivalent welded interface, $\mathbf{R}_0 = -\left(\boldsymbol{\Xi}^-\right)^{-1}\boldsymbol{\Xi}^+$, and an imaginary part as induced by the slip:

$$\mathbf{R} \approx \mathbf{R}_0 + i\mathbf{R}_{SLIP}, \tag{7}$$

with

$$\mathbf{R}_{SLIP} = \omega\left(\boldsymbol{\Xi}^-\right)^{-1}\mathbf{Z}\left(\mathbf{L}_{2,I}^+ + \mathbf{L}_{2,I}^-\mathbf{R}_0\right). \tag{8}$$

The imaginary part of the reflection matrix $\mathbf{R}_{SLIP}$ can thus be used directly as an indicator for the interface compliance matrix $\mathbf{Z}$.

EXAMPLES

In the first example we model seismic P-wave reflection coefficients at a frequency of 30 Hz at four different fault interfaces, with properties as given in Table 1. The faults are at an angle of 60 degrees with respect to the horizontal plane and they are sandwiched between Vertical Transverse Isotropic (VTI) shale and sandstone. We choose a new coordinate system such that the fault is aligned with the horizontal plane, implying rotation of the media's stiffnesses over 60 degrees, using Bond transformation matrices (Auld, 1973). Figure 1 shows the imaginary parts of the reflection coefficients with varying angles of incidence, evaluated with equation (5) (colored solid lines) as well as with low frequency approximation (8) (black dashed lines). Note that the effects of slip are best exposed at normal incidence and that the low frequency approximations match closely the exact responses. Next we evaluate the imaginary parts of the reflection intercepts with varying frequencies – see Figure 2. The low frequency approximation matches closely the exact results for all 4 interfaces for frequencies up to 60 Hz.

Interface	A	B	C	D
normal compliance ($10^{-11} m/Pa$)	0.0	2.3	4.6	7.0
tangential compliance ($10^{-11} m/Pa$)	8.1	8.1	8.1	8.1
coupling compliance ($10^{-11} m/Pa$)	0.0	0.0	0.0	0.0

Table 1: Properties of interfaces A, B, C and D.

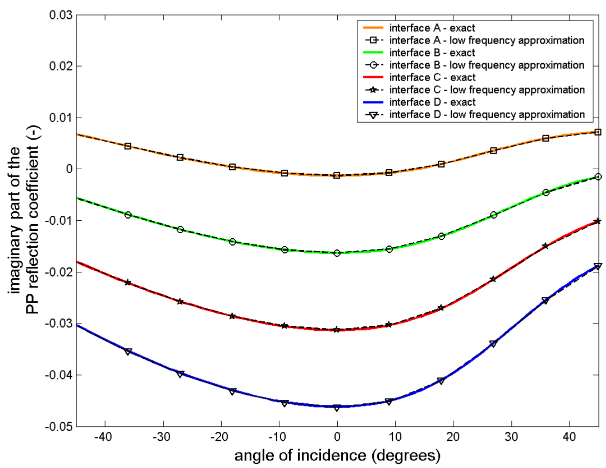

Figure 1: Imaginary parts of the reflection coefficients of interfaces A, B, C and D with varying angles of incidence; solid lines are exact; dashed lines are in the low frequency approximation

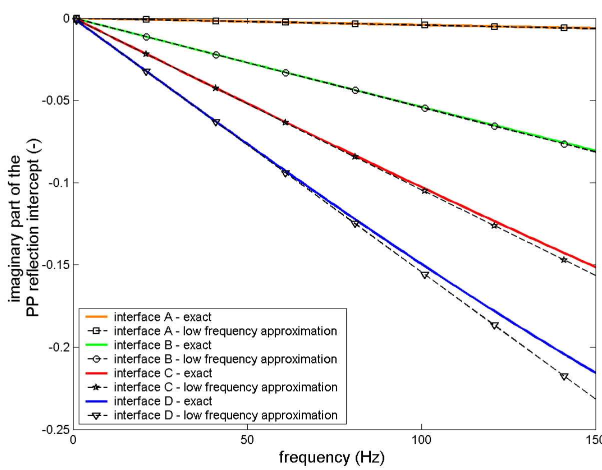

Figure 2: Frequency dependence of the imaginary parts of the reflection intercepts of interfaces A, B, C and D; solid lines are exact; dashed lines are in the low frequency approximation.

In another example we model PS converted reflection coefficients at a frequency of 30 Hz at four fault interfaces with different coupling compliances; their properties being given in Table 2. The faults are sandwiched between isotropic shale with P-wave velocity $\alpha_I = 2730 m/s$, S-wave velocity $\beta_I = 1240 m/s$ and density $\rho_I = 2350 m/s$ and isotropic sandstone with P-wave velocity $\alpha_{II} = 2020 m/s$, S-wave velocity $\beta_{II} = 1230 m/s$ and density $\rho_{II} = 2130 m/s$. Figure 3 shows the imaginary parts of the PS converted reflection coefficients with varying angles of incidence. The low frequency approximations match closely the exact responses. The 'unusual' PS conversion at normal incidence might be used to estimate the coupling compliance directly through equation (8).

Interface	E	F	G	H
normal compliance ($10^{-11} m/Pa$)	12	12	12	12
tangential compliance ($10^{-11} m/Pa$)	15	15	15	15
coupling compliance ($10^{-11} m/Pa$)	0	4	8	12

Table 2: Properties of interfaces E, F, G and H.

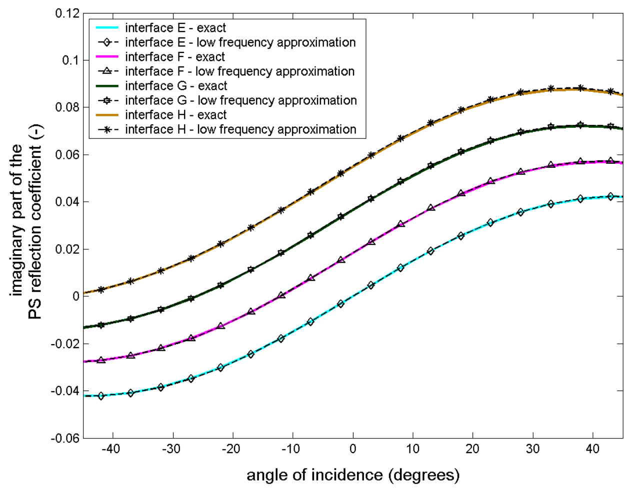

Figure 3: Imaginary parts of the PS reflection coefficients of interfaces E, F, G and H with varying angles of incidence; solid lines are exact; dashed lines are in the low frequency approximation.

CONCLUSION

We used linear slip theory to derive general expressions for seismic reflection coefficients at non-welded interfaces, such as faults or fractures, between general anisotropic media and approximated these coefficients at low frequencies (< 60 Hz). We showed that the real parts of these approximated reflection coefficients match the responses of the equivalent welded interfaces, whereas the slip induced parts are purely imaginary in this regime.

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