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## Effect of Cooperative Fuzzy c Means Clustering on Estimates of Three Parameter AVA Inversion

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### Summary

We determine the variation in degree of model fitness with respect to a true model based on Amplitude variation with angle (AVA) methodology by a synthetic Gas hydrate model, using cooperative Fuzzy c means clustering, constrained with a rock property model. When a homogeneous starting model is used, with only traditional least square optimization scheme for inversion, the variance of the density parameter is found to be high in contrast with compressional and shear velocity. Tests with cooperative inversion using Fuzzy c means with 4<sup>th</sup> order cluster with inclusion of a least square damping technique, constrained with a derived empirical relationship based on rock properties model, the results shows improved stability, model fitness and variance for all the three parameters in comparison with the standard inversion alone.

### Introduction

Several elastic parameters like P-wave velocity (VP), S-wave velocity (VS) and density ( $\rho$ ) across an interface can be found out using AVA (amplitude variation with angle) information. The AVA is mainly governed by Zoeppritz equation (Zoeppritz, 1919). The strict mathematical expression of the Zoeppritz equation is exceptionally long. As a result, it makes it difficult to notice how reflection amplitude varies if a rock property is changed slightly. Many approximations to the Zoeppritz equation is made by several scientists (Aki and Richards 1980). These equations are focussed to give an intuitive understanding of how these amplitudes are related to various physical parameters that is valid for small angle of incidence and low impedance contrast.

### FCM Cluster analysis with damped least square technique

The non linear problem can be stated as follows:

$$R_{pp} = Q(1,1)$$

$$R_{pp} = f\left(\frac{\Delta V_s}{V_s}, \frac{\Delta V_p}{V_p}, \frac{\Delta \rho}{\rho}, \theta\right) + \varepsilon \quad (1)$$

Based on equation (1), we propose an inversion procedure to estimating Vs, Vp and density reflectivity. The basic assumptions made are that the earth has approximately horizontal layers at each common depth point, and that each layer is described by the three parameters

$$V_s, V_p, \rho$$

We use the Full Zoeppritz equation to compute  $R_{pp}(\theta)$ . The Full Zoeppritz equation is given by:

(2)



$$Q = P^{-1}R$$

$$P = \begin{bmatrix} -\sin\theta_1 & -\cos\phi_1 & \sin\theta_2 & \cos\phi_2 \\ \cos\theta_1 & -\sin\phi_1 & \cos\theta_2 & -\sin\phi_2 \\ 2\rho_1 V_{s1} \sin\phi_1 \cos\theta_1 & \rho_1 V_{s1} (1 - 2(\sin\phi_1)^2) & 2\rho_2 V_{s2} \sin\phi_2 \cos\theta_2 & \rho_2 V_{s2} (1 - 2(\sin\phi_2)^2) \\ -\rho_1 V_{p1} (1 - 2(\sin\phi_1)^2) & \rho_1 V_{s1} \sin(2\phi_1) & \rho_2 V_{p2} (1 - 2(\sin\phi_2)^2) & -\rho_2 V_{s2} \sin(2\phi_2) \end{bmatrix}$$

$$R = \begin{bmatrix} \sin\theta_1 & \cos\phi_1 & -\sin\theta_2 & -\cos\phi_2 \\ \cos\theta_1 & -\sin\phi_1 & \cos\theta_2 & -\sin\phi_2 \\ 2\rho_1 V_{s1} \sin\phi_1 \cos\theta_1 & \rho_1 V_{s1} (1 - 2(\sin\phi_1)^2) & 2\rho_2 V_{s2} \sin\phi_2 \cos\theta_2 & \rho_2 V_{s2} (1 - 2(\sin\phi_2)^2) \\ \rho_1 V_{p1} (1 - 2(\sin\phi_1)^2) & -\rho_1 V_{s1} \sin(2\phi_1) & -\rho_2 V_{p2} (1 - 2(\sin\phi_2)^2) & \rho_2 V_{s2} \sin(2\phi_2) \end{bmatrix}$$

(3)

The parameter vector to be optimized comprises n Vs values, followed by n Vp values and n density values where n is the number of layer such as:

$$P = [V_{S1} \dots V_{Sn}, V_{P1} \dots V_{Pn}, \rho_1 \dots \rho_n]$$

(4)

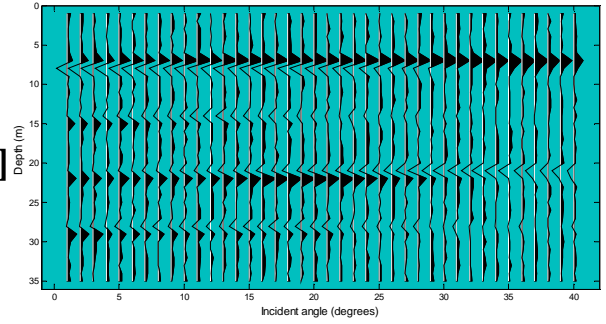
From a given starting model  $P_{start}$ , the zero-offset Vs, Vp and density reflectivity at an interface i can be calculated as:

$$\frac{\Delta V_s}{V_s} = \frac{V_s^i - V_s^{i-1}}{(V_s^i + V_s^{i-1}) / 2}$$

(item for Vp and Vs)  
(5)

By substituting equations (5) into equation (1), we can compute the reflection coefficient  $R_{pp}(\theta)$  for each angle and at each layer boundary. A synthetic offset seismic gather can be calculated by convolving the reflection coefficients  $R_{pp}(\theta)$  with predetermined wavelets.

Fuzzy c-means (FCM) is a data clustering technique wherein each data point belongs to cluster to some degree that is specified by a membership grade. It provides a method that shows how to group data points that populate some multidimensional space into a specific number of different clusters.



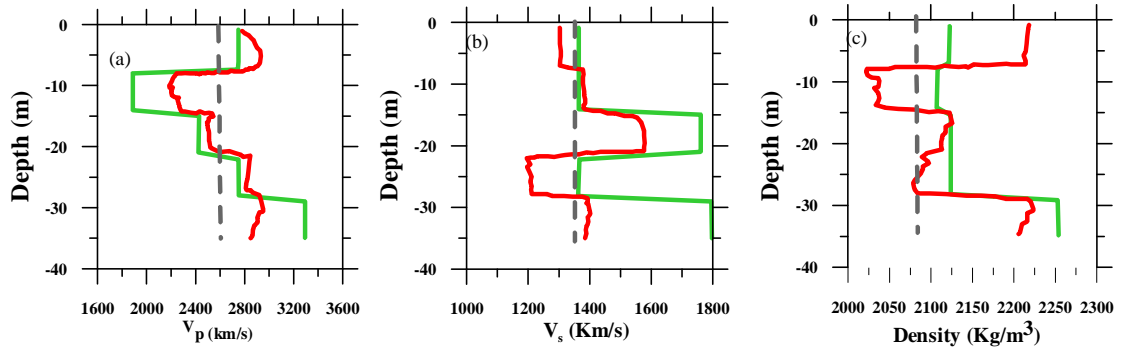
**Synthetic Data**

The input model for our synthetic example consists of five heterogeneous layer boundaries with acoustic impedance contrasts.

Fig1. Forward modeled synthetic seismic data used for Fuzzy c means clustering.

### Results of the Standard Inversion

The non linear inversion does not find the exact parameter values but the results are satisfactory. Density is the least well determined parameter as for the linear inversion. When adding noise to the data (S/N=4), the unconstrained non linear inversion is not destabilized as the linear inversion. The results in figure (2) are close to the results of the noise free case, except for the density estimate which loses accuracy. We conclude from the synthetic modeling that the non linear inversion is more robust than the linear inversion. The non linear inversion is little affected by the noise level in the data.



**Figure 2.** Results of standard inversion with least square minimization: Green line indicates true model, red line indicates inverted model and dashed line indicates the initial guess model.

### Results of using the fuzzy c-means clustering approach

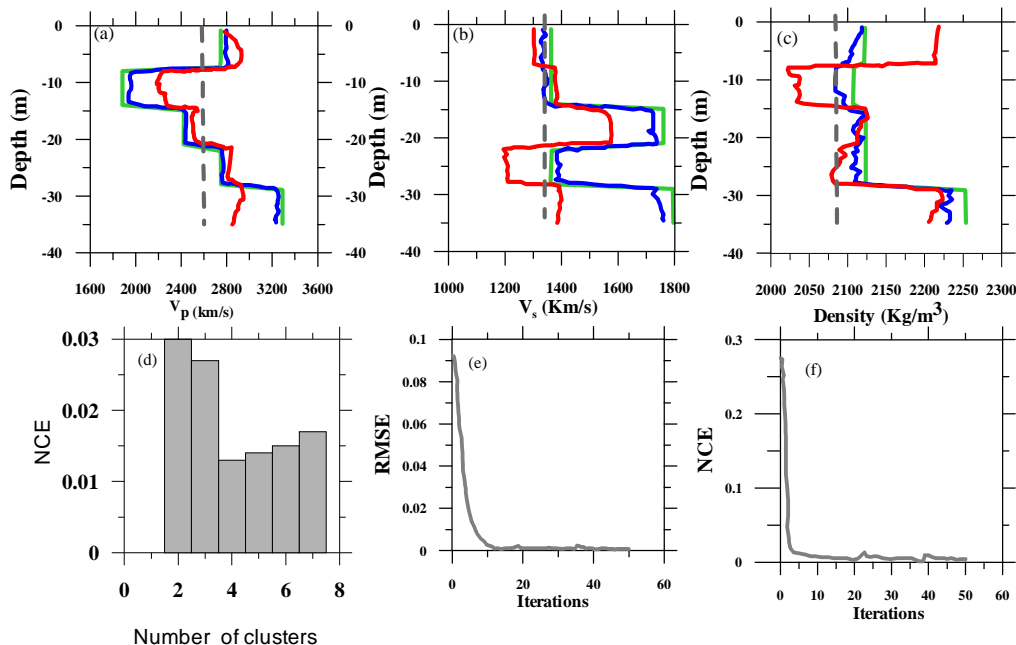
When a cooperative inversion procedure is adopted, first with the starting model as a homogenous model, then we found that the cooperative fuzzy c means gives a better fit to the model than the simple inversion with least square, and then with least square inverted output as a starting model, then the convergence is better, with NCE and RMS values going on to minimum. The results in figure (3) based on fuzzy c means are close to the results of the true model.

The velocity models resulting from the single inversion and cooperative inversions are shown in Figure 2 and Figure 3. When analyzing the complete results, overall variability as well as the averaged velocity values each layer, are indeed

comparable to the corresponding characteristics of the input model used. This is shown here in Figure 2, with poor model fit for single inversion and a better model fit for cooperative inversion, when the starting model is the least square inverted model. However, as the smoothness constraints influence the inversion process, the original clustered character of the velocity relation is not exactly reconstructed. In Figure 3, it is shown that minimum NCE values are obtained for cluster 4.

According to Occams Razor, we have chosen the four cluster solution that is the simplest model explaining both data sets—minimum rms errors and satisfying our statistical criterion—minimum NCE value.

We conclude from the synthetic modelling that the cooperative fuzzy c means non linear inversion is more robust than the conventional single optimization scheme algorithms for inversion. Since the whole analysis is done with Signal to noise ratio 4, it is found that results are little affected by the noise level in the data. The percentage error for the parameters after cooperative fuzzy c means is 2%, 2.5%, 9.3% for  $V_p$ ,  $V_s$  and density respectively.



**Figure 3.** (a) to (c), Fuzzy C means inversion after least square inverted results are given as input

model. The blue line indicates fuzzy inverted result, red the least square inverted result. (e) indicates the



minimum clusters for a lowest normalized classification entropy (f) RMSE error shows the parameters are obtained, when the error goes to minimum (g) NCE values again indicates that results are obtained when the NCE values also goes to minimum.

## Conclusions

We have reassessed the nature of model fitness and stability issue for a AVA based scheme in inverting compressional, shear velocity and density. We used a cooperative fuzzy c means cluster analysis that is less affected by bias than other methods of inversion scheme. Our results shows that the ability to resolve all the three parameters well, which was hitherto been not resolved. The simplicity and flexible approach of the scheme, suggests the possibility for 2D and 3D problems for hydrocarbon exploration based on AVA.

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