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A FORTRAN code for quantitative interpretation of self potential anomalies

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Summary

A formula has been used based on least-squares minimization process to compute nature of source geometry and parameters from SP anomalies. A Flow-chart is described to implement the algorithm. The FORTRAN code has been perfected to develop an iterative process for picking up origin of the anomaly. The code has been tested over simulated SP anomaly with and without random noise over all three different models. These analyses reveal that the methodology can tolerate random error of 30% with acceptable errors in computed parameters. The stability of the method is evaluated by varying polarization angle and depth. An effect of depth to spacing ratio is also studied. Further applicability has been demonstrated through two field examples adopted from published literatures.

Introduction

Self potential, naturally occurring potential difference has application in search of mineralised bodies. Though several methods of quantitative interpretation have been developed like 'curve matching' by Murty and Haricharan(1985),'derivative analysis' by Abdelrahman et al.(2003),'Modelling and Inversion' by Shi and Morgsn(1996), enhanced local wave number technique by Srivastava and Agarwal(2009), etc., but most of these are greatly influenced by the noise in the measured data. The present method developed by El-Araby (2003) may handle the random errors and noisy data more effectively than others. A least squares minimization approach (El-Araby, 2003) is implemented to determine the shape factor using all data points in the self potential anomaly. The problem is transformed into solving a nonlinear equation for shape factors to calculate other parameters like depth, polarization angle, etc. The only requirement of this method is to assign the origin of the source model on profile data. In the present approach I find out the optimum origin, data interval and graticule spacing (integer multiple of data spacing) by using an iterative approach to arrive at the best model with lowest RMS error. Synthetic data were generated with and without random noise (upto 30%) and applied to the code developed. This study indicates that the method provides model parameters in error less than 5%. The applicability of algorithm is further tested on field data over Suleymankoy and Weiss from Turkey with consistent results.

Theory

The general self-potential (SP) anomaly expression produced by most polarized structures along a principal profile over the body is given by the following equation at a point P(x_i, z) (Figure 1)

$$v(x_i, z, \theta, q) = k\,\frac{x_i.\cos\theta + z.\sin\theta}{(x_i^2 + z^2)^q} \qquad ...(1)$$

For (2N+1) points, x_i, i= -N,…..-1,0,1,……..N

Where

z is the depth,

h is the polarization angle between the axis of polarization and the horizontal,

x_i is a discrete point along x-axis

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q is the shape-factor which defines the geometry of the source body,
k is the electric current dipole moment.

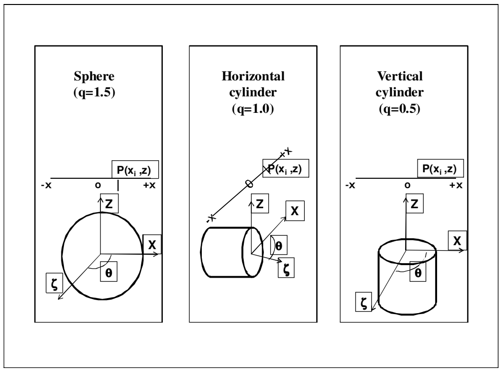

Figure 1: The target models showing coordinate system and direction of polarization

The shape-factors for a sphere, a horizontal cylinder and a semi infinite vertical cylinder are 1.5, 1.0 and 0.5, respectively.

Replacing k (unknown) by v(0), the self potential value at the origin (known) ,we get

$$v(x_i, z, \theta, q) = v(0).z^{2q-1}\frac{x_i.\cot\theta + z}{(x_i^2 + z^2)^q} \qquad ...(2)$$

Considering two observation point at x_i=-s and x_i =s and replacing depth (z) & polarization angle (θ) (unknown) by v(s) and v (-s) (known) we get an equation of only one unknown (q) as

$$v(x_i, q) = v(0).w(x_i, q) \qquad ...(3)$$

Where

$$w(x_i, q) = s^{2q-1}\frac{x_i.P + s.F}{[x_i^2 + F^{1/q}(s^2 - x_i^2)]^q}$$

$$F = \frac{v(s) + v(-s)}{2.v(0)} \quad , \quad P = \frac{v(s) - v(-s)}{2.v(0)}$$

The unknown q can be obtained by minimizing

$$\phi(q) = \sum_{i=-N}^{N}[Y(x_i) - v(0)w(x_i, q)]^2 ....(3)$$

where, $Y(x_i)$ = observed sp anomaly at x_i

Thus, the final equation is

$$q = \frac{\sum_{i=-N}^{N} v(0).w^2(x_i, q)\{\frac{(s^2 - x_i^2).F^{1/q}.\ln(F)}{x_i^2 + F^{1/q}(s^2 - x_i^2)}\}}{\sum_{i=-N}^{N} Y(x_i).\frac{d}{dq}w(x_i, q) - \sum_{i=-N}^{N} v(0).w^2(x_i, q).\ln\{\frac{s^2}{x_i^2 + F^{1/q}(s^2 - x_i^2)}\}} \qquad ...(4)$$

Which transforming into an iterative form becomes

$$q_c = f(q_i) \qquad ...(5)$$

Where q_c is the calculated revised shape factor and q_i is the initial shape factor.

Software (FORTRAN code)

The software corresponding to the method described above has been generated using "FORTRAN POWER STATION". The flow chart for the program is given below.

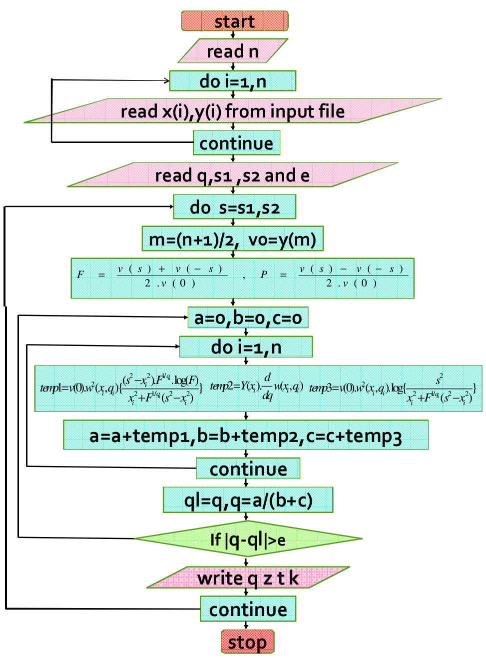

Figure 2: Flowchart of the algorithm.

Following this flow chart a FORTRAN code has been developed.

Synthetic data analysis

The above mentioned program was compiled successfully but it is very necessary to test its ability to solve the interpretational problems. The first step to this approach is to work with synthetic data. Synthetic data of the spherical source model, a horizontal cylindrical source model and a vertical cylindrical source model were generated using the programming corresponding to equation no. (1) for different model parameter and the feasibility of the method is demonstrated.

Spherical source model

Self potential anomaly for buried spherical source has been calculated for the following parameter (polarization angle (θ) =30 degrees, dielectric constant (k) = -100) at different depth (z = 2, 2.5, 3, 3.5, 4, 4.5 and 5 unit) (figure 3) and then using that anomaly as an input of the developed software the corresponding model parameters have been computed. When the computed and actual model parameters are compared they show identical values (Table 1).

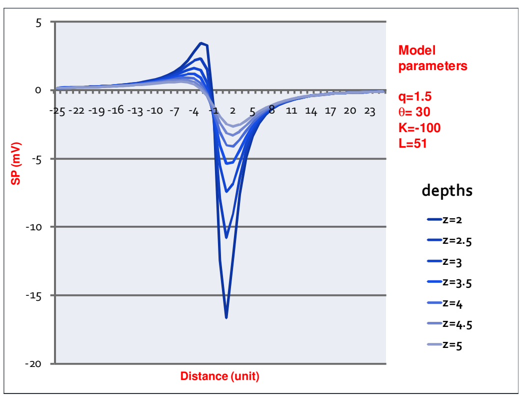

Figure 3: Synthetic data (noise free) for spherical source at different depth.

depth	q	Z(unit)	q(in degree)	K
2	1.49999	1.99998	30.00019	-99.99673
2.5	1.49999	2.49997	30.00029	-99.99500
3	1.49998	2.99994	30.00053	-99.98978
3.5	1.49997	3.49990	30.00072	-99.98476
4	1.49996	3.99986	30.00086	-99.97998
4.5	1.49995	4.49981	30.00104	-99.97393
5	1.49994	4.99975	30.00122	-99.97182

Table 1: Outputs for model parameters, Model: Sphere, q=1.5, q = 30, K= -100, L=51, s=14.

Horizontal cylindrical source model

Similarly, synthetic self-potential anomaly for horizontal cylindrical source has been computed for polarization angle (θ) =30 degree, dielectric constant (k) = -100 and different depth (z = 2, 2.5, 3, 3.5, 4, 4.5 and 5 unit) (Figure 4) and the output of the execution were compared. In this case also the calculated and actual model parameters are same (Table 2).

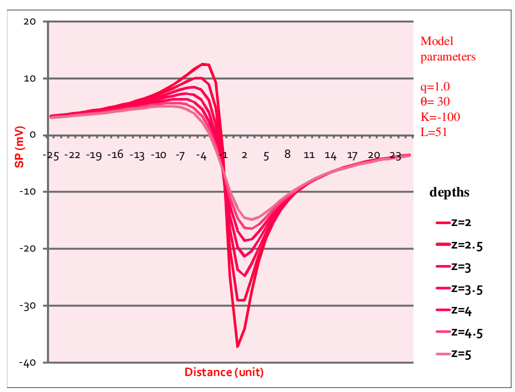

Figure 4: Synthetic data (noise free) for horizontal cylindrical source at different depth.

depth	q	Z(unit)	q(in degree)	K
2	1.00000	2.00000	29.99999	-100.00030
2.5	1.00001	2.50002	29.99976	-100.00270
3	0.99999	2.99995	30.00039	-99.99519
3.5	0.99998	3.49990	30.00068	-99.99067
4	0.99998	3.99988	30.00076	-99.98846
4.5	0.99997	4.49983	30.00094	-99.98460
5	0.99996	4.99978	30.00109	-99.98082

Table 2: Outputs for model parameters, Model: Horizontal cylinder, q=1.0, □= 30, K= -100, L=51, s=14.

Vertical cylindrical model

The computed model parameters for the synthetic data generated by a vertical cylindrical model (Figure 5) also follow the same path as in case of the other two common geometrical shapes discussed above.

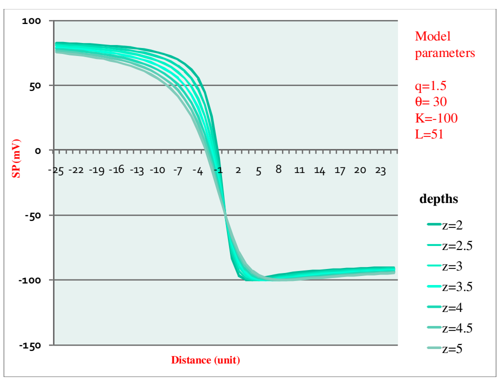

Figure 5: Synthetic data (noise free) for vertical cylindrical source at different depth

depth	q	z(unit)	q(in degree)	K
2	0.50000	1.99996	30.00046	-99.99805
2.5	0.50000	2.50000	30.00004	-99.99980
3	0.50000	3.00000	29.99997	-100.00020
3.5	0.50000	3.50004	29.99939	-100.00180
4	0.50001	4.00010	29.99939	-100.00390
4.5	0.49999	4.49939	30.00117	-99.99200
5	0.49999	4.99979	30.00106	-99.99224

Table 3: outputs for model parameters, Model : Vertical cylinder , q=0.5, □□= 30, K= -100, L=51, s=14.

Analysis of synthetic data with random noise

Although the present method shows excellent results in case of noise free synthetic data but as the world of signal is no where noise free in practical consideration, so a random noise of 5% was introduced in each of the cases mentioned above.

Spherical source model

The self potential anomaly for a spherical source model mentioned earlier with 5% random noise is shown (Figure 6) below. Then using the programming the model parameters were calculated and the percentages of errors in each model parameters were plotted against depth. It is clear from the plot (Figure 7) that the error percentages in model parameters are less than 5% at each depth.

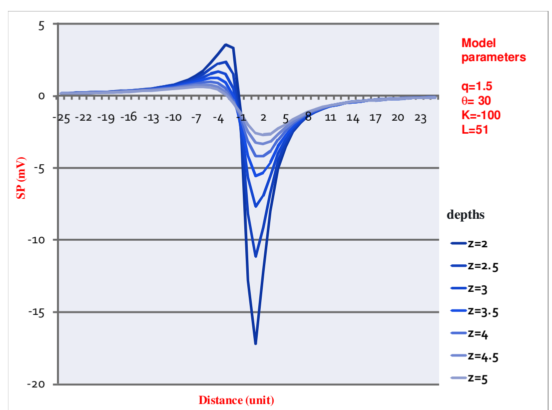

Figure 6: Synthetic data (with 5% noise) for spherical source at different depth

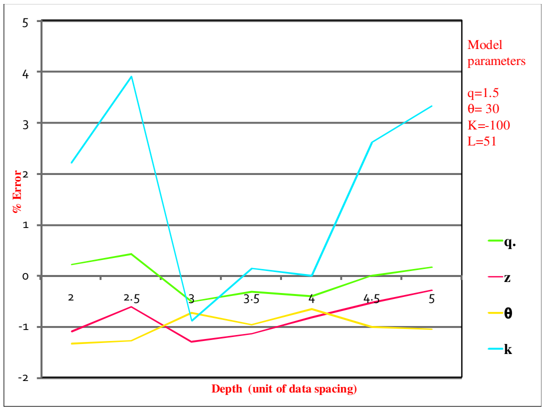

Figure 7: Error response in model parameters (best s) for spherical source with 5% random errors.

Horizontal cylindrical source model

Similarly, model parameters for the self potential anomaly for a horizontal cylindrical source with 5% random noise are shown in the graph (Figure 8). The error response curves in model parameter estimation indicate the applicability of this method in horizontal cylinder also.

Vertical cylindrical source model

The synthetic self potential data for vertical cylindrical source model with 5% random noise were also computed and the corresponding percentages of error in model parameters are shown (Figure 9). The result gives the proof that the vertical source model do not contradict the success of present method.

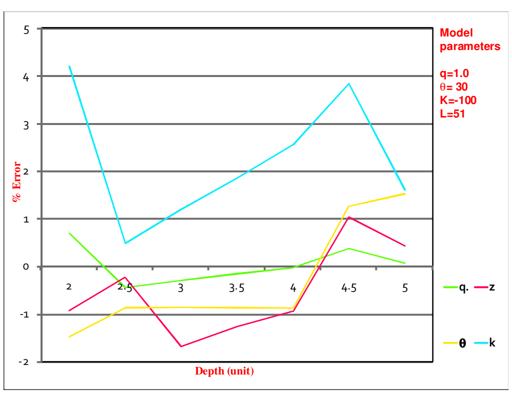

Figure 8: Error response in model parameters (best s) for horizontal cylindrical model with 5% random noise.

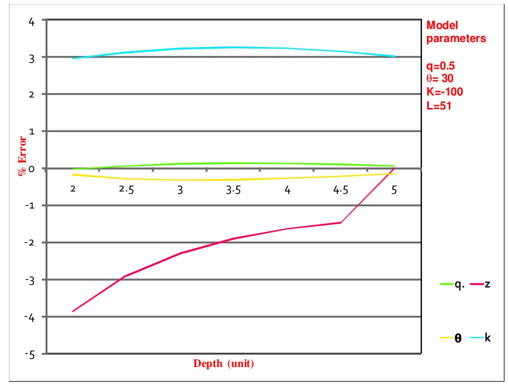

Figure 9: Error response in model parameters (best s) for vertical cylindrical source model with 5% noise.

Stability of the method for polarization angle

Any of the three above mentioned model was selected (say horizontal cylinder). Then synthetic data were generated (Figure 10). Then corresponding model parameters were estimated and error percentage in model parameters were calculated and plotted against polarization angle (Figure 11).

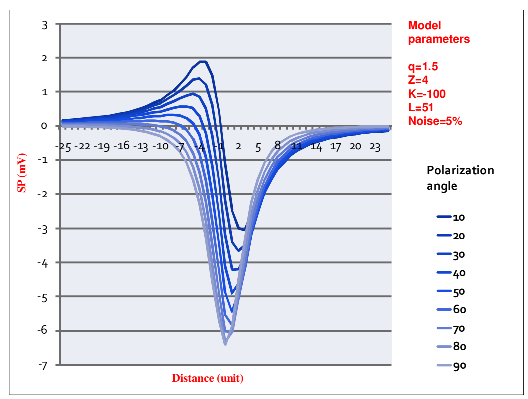

Figure 10: Synthetic data (with 5% noise) for spherical source at different polarization angle

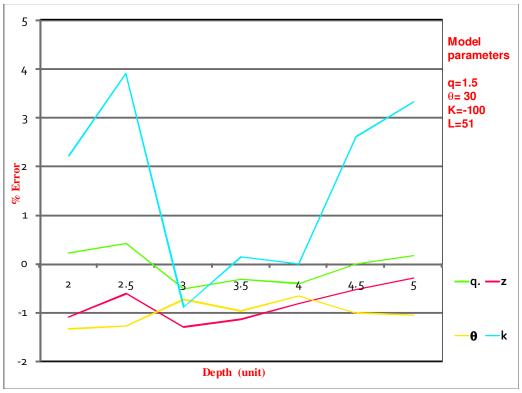

Figure 11: Error response in model parameters as a function of polarization angle.

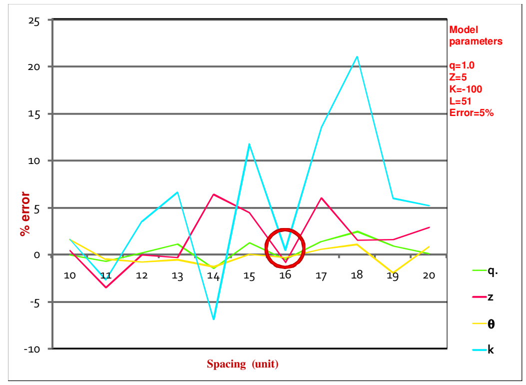

Figure 13: Error response in model parameters as a function of spacing (s)

Choice of the spacing S

Again arbitrarily one model was selected (say horizontal cylinder for depth (z)= 5, polarization angle(θ) =30 degree, dielectric constant(k) = -100,and noise = 5%) and the set of expressions of present methods were executed for different S (10 to 20)(Figure 12). It is crystal clear from the error response in model parameters(Figure 13) that there is a value of S (16) where all estimated model parameters comes very close to the actual one i.e. error percentage reduces simultaneously.

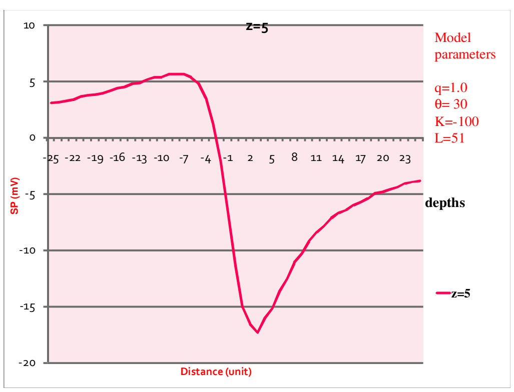

Figure 12: Synthetic data (with 5% noise) for horizontal cylindrical source at depth=5 units

Effect of increasing noise percentage in the synthetic data

A buried spherical source model's self potential response was recalculated for different percentage of random noise (5% to30%)(Figure 14). After estimating the values of shape factor, depth, polarization angle, and dielectric constant their percentage errors were plotted separately within a figure (Figure 15). The four graphs corresponding to the four model parameters illustrates that the error percentage does not exceed 5 marks even when noise percentage is 30.

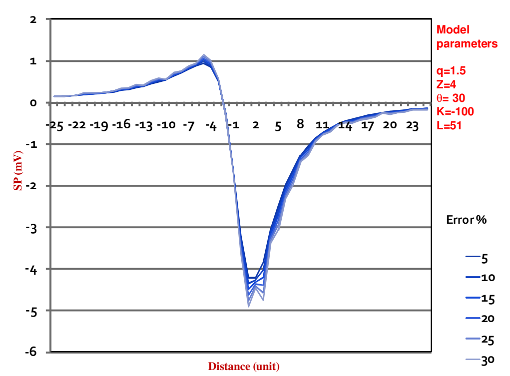

Figure 14: Synthetic data for spherical source applying different

percentage of random noise

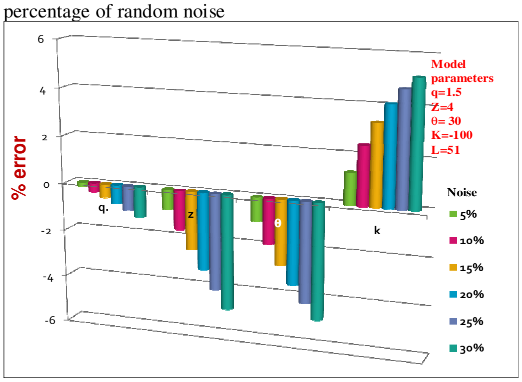

Figure 15: Error response in model parameters with increase in error percentage in synthetic noisy data for spherical source.

Field examples

To determine the practical utilities of the present method a field data of Suleymankoy, Turkey was also analyzed. Here, the observed data are set as an input of the present programming and the outputs are calculated. Then the RMS errors (defined in equation (6)) were calculated to have an idea about the closest fitted outcome.

The RMS error can be defined as

$$\text{RMS}=\sqrt{\frac{\sum_{i=-N}^{N}[Y(x_i)-V(x_i,z,\theta,k)]^2}{2N+1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (6)$$

Where Y (x_i) is the observed anomaly and V is the computed anomaly.

Suleymankoy anomaly

The suleymankoy self potential anomaly at Ergani in Turkey of profile length 264m was digitized at a spacing of 6m using grapher and is shown in the figure (Figure 17) below.

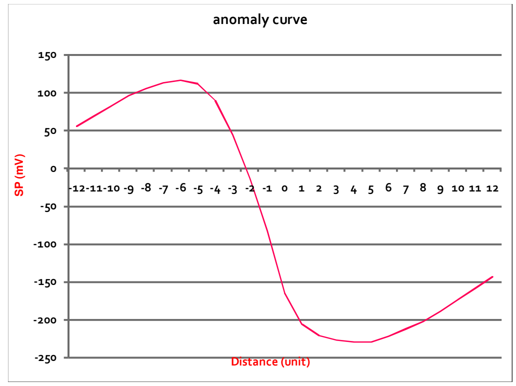

Figure 16: Sulleymonkoy anomaly, Ergani, Turkey; Profile length= 25 units (taking 1 unit=6m)

The result obtained for different spacing S is shown in tabular form (Table 4) and for the origin at 72m the minimum RMS error (23.22) was obtained. Then the corresponding model parameters are used to generate its self potential response and it was compared with the actual anomaly.

s	q	z	q	k	RMS err
5	0.806	3.13	29.13	-682.76	24.080
6	**0.859**	**3.62**	**27.51**	**-900.74**	**23.213**

Table 4: Output results and corresponding RMS errors

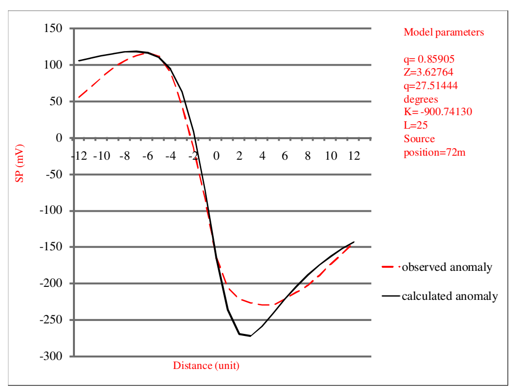

Figure 17: Comparison of calculated and measured sp anomaly

Finally the result was compared with the result obtained from the method of Srivastava and Agarwal (2009). According to them the source was cylindrical at a depth of 28.9m and source position was at 64.1m. Now the the present algorithm says that the source is cylindrical at a depth of 21.76m with source position at72m and polarization angle 27.5 degree. Thus the result obtained is close to obtained by other methods.

Weiss Anomaly

A self potential profile of Weiss anomaly (Figure 18) was digitized at 25 points at an interval of 7.7 m. The model parameters obtained from the algorithm are compared with the result obtained by other methods (Table 5). An appreciable result is visible in the table.

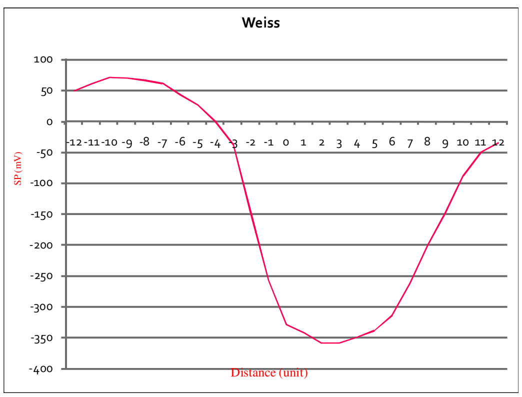

Figure 18: Weiss Anomaly, Ergani, Turkey

Name of the method	Depth	Polarization angle	Shape factor
Yungul(1950)	53.8m	40^o	1.5
Bhattacharya & Roy(1981)	54m	30^o	1.5
Abdelrahman(1996)	52.9m	35.3^o	1.5
Present method	45.7m	34.2^o	1.67

Table 5: Comparison of model parameters of Weiss anomaly

Present Development: After realizing the importance of choosing right spacing and source position the algorithm was developed to test for a range of source position and Spacing. Further an interpolation algorithm was introduced into the software to make the spacing between two consecutive stations in user's control

Conclusion

The present code gives exact values for synthetic noise free data for all the three types of model. Also the error percentage in model parameters is always less than 5% when using synthetic anomaly with 5% random noise for all the three types of models. This software is stable for a wide range of polarization angle, depth and spacing. There exists a value of S for which all the estimated model parameters shows minimum error percentage simultaneously. Further the error percentage in model parameters does cot exceeds the mark 5% even when the percentage of random noise reaches 30%. Finally the field examples of Suleymankoy and Weiss give a good response to the software and further extend its applicability to real field problems.

References:

El-Araby, H.M., 2004. A new method for complete quantitative interpretation of self-potential anomalies. Journal of Applied Geophysics 55, 211-224.

Srivastava, S., Agarwal, B.N.P., 2009. Interpretation of self-potential anomalies by Enhanced Local Wave number technique, Journals of Applied Geophysics,doi:10.1016/j.jappgeo.2008.11.011

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