Support-operator method based frequency-domain acoustic-wave modeling in a transversely isotropic medium with tilted symmetry

Iktesh Chauhan*[1], Sujith MS[1,2] & Rahul Dehiya[1]
[1]*Department of Earth and Climate Sciences, Indian Institute of Science Education and Research (IISER) Pune*
[2] *University of Texas at Austin*
∗iktesh.chauhan@students.iiserpune.ac.in

Keywords

Support operator method, *Thomsen's Parameter*, *VTI*, *TTI*, *absorbing boundary conditions*

Abstract

We developed a new algorithm to simulate frequency-domain acoustic-wave response in a transversely isotropic medium with tilted symmetry. The algorithm uses a support-operator finite-difference method for modeling. This method constructs a finite-difference scheme on a logically rectangular grid for second-order elliptic equations with general coefficient matrices and boundary conditions. The medium's properties are described as P-wave velocity on the symmetric axis, density, Thomsen's anisotropic parameters (epsilon and delta), and the tilt angle. To evaluate the accuracy of the scheme, we conducted several synthetic experiments. The results suggest that the developed algorithm simulates the P-wave solution and the fictitious S-wave mode, as reported in the literature. The findings of numerical experiments indicate that the algorithm, devised using a support-operator finite-difference method, simulates tilted anisotropic acoustic waves in the frequency domain accurately and efficiently.

Introduction

In exploration seismology, anisotropy subsurface leads to the directional dependence of seismic wave propagation caused by the properties in subsurface material. Modeling the response of the anisotopic model is vital in the exploration of seismology for accurate velocity model building [Li et al., 2008], amplitude interpretation and reservoir characterization [Besheli et al., 2005]. To model the propagation of seismic waves in anisotropic media, researchers commonly employ numerical methods such as finite difference, finite element, and pseudospectral methods [Igel et al., 1995, Carcione, 1999]. When simulating the propagation of P-waves in vertical transversely isotropic (VTI) and orthorhombic anisotropic media, an approach proposed by Alkhalifah [1998, 2003] involves the solution of a modified acoustic wave equation. This equation simplifies computational requirements by setting the vertical S-wave velocity to zero in the dispersion relation. This acoustic VTI wave equation reduces computational costs, as it eliminates the need to solve the more complex anisotropic elastic wave equations. Alkhalifah [1998] demonstrated that this approach yields a kinematically accurate approximation of P-wave propagation.

The simulation of wave propagation can be implemented either in the time domain [Mehra et al., 2012, Gao et al., 2018] or frequency domain [Williamson and Pratt, 1995, Hustedt et al., 2004, Plessix, 2007]. Frequency-domain modeling is computationally efficient, better suited for frequency-dependent behavior and long-distance wave propagation, and allows greater control over wave sources in acoustic wave modeling. Researchers have significantly advanced anisotropic acoustic wave algorithms for Vertical Transverse Isotropic (VTI) and Tilted Transverse Isotropic (TTI) media in the past few years. Zhou et al. [2006] introduced an algorithm for VTI media, while Operto et al. [2009] focused on frequency-domain modeling of visco-acoustic wave propagation through a TTI media.

The study presents a novel approach to modeling two- dimensional anisotropic acoustic waves by employing the support-operator method. Instead of working in the time domain, we implement the equation proposed by [Zhou et al., 2006] in the frequency domain. The anisotropic medium is characterized by several parameters, including the P-wave velocity along the symmetry axis, density, anisotropic parameters ϵ and δ, and the tilt angle. The support-operator method is a mathematical technique used in numerical simulations to partial differential equations [Shashkov, 1996]. The support operator method offers a unique approach to solving partial differential equations in complex geometrical scenarios. It achieves this by providing the divergence of gradient, **DIV GRAD** of a function ( ), which enables efficient handling of intricate geometric structures. To ensure effective absorption of wave reflections at the boundaries, we implement perfectly matched layers (PML) as the absorbing boundary conditions, as proposed initiallyby [Berenger, 1994].

Theory

For TTI media, we begin with a modification of the 2D acoustic wave equation of Zhou et al. [2006]:

$$\frac{1}{\kappa}\frac{\partial^2 p}{\partial t^2} - (1+2\delta)Hp - H_0 p = (1+2\delta)Hq \qquad (1)$$

$$\frac{1}{\kappa}\frac{\partial^2 q}{\partial t^2} - 2(\epsilon-\delta)Hq = 2(\epsilon-\delta)Hp \qquad (2)$$

Where H and H0 are;

$$H = \cos^2\theta_0 \frac{\partial}{\partial x} b \frac{\partial}{\partial x} + \sin^2\theta_0 \frac{\partial}{\partial z} b \frac{\partial}{\partial z} - \frac{\sin 2\theta_0}{2}\left(\frac{\partial}{\partial x} b \frac{\partial}{\partial z} + \frac{\partial}{\partial z} b \frac{\partial}{\partial x}\right)$$

$$H_0 = \sin^2\theta_0 \frac{\partial}{\partial x} b \frac{\partial}{\partial x} + \cos^2\theta_0 \frac{\partial}{\partial z} b \frac{\partial}{\partial z} + \frac{\sin 2\theta_0}{2}\left(\frac{\partial}{\partial x} b \frac{\partial}{\partial z} + \frac{\partial}{\partial z} b \frac{\partial}{\partial x}\right)$$

where p is the pressure wavefield, q is the auxiliary pressure wavefield introduced by Zhou et al. [2006] to recastthe fourth-order equation proposed by Alkhalifah [1998] into the system of second-order equations (1) and (2), κ is the bulk modulus along the symmetry axis, and b is the inverse of mass density, termed buoyancy. The values δ and ϵ are dimensionless parameters defining the anisotropy introduced by Thomsen and θ_0 is the angle of the symmetry axis with the z-axis.

By transforming equations (1) and (2) to the Fourier domain, we obtain the acoustic wave equation in the frequency domain as,

$$\frac{\omega^2}{\kappa} p - (1+2\delta)Hp - H_0 p = (1+2\delta)Hq \qquad (3)$$

$$\frac{\omega^2}{\kappa} q - 2(\epsilon-\delta)Hq = 2(\epsilon-\delta)Hp \qquad (4)$$

where ω is the angular frequency, we assume that the θ_0 changes relatively smoothly with both x- and z-directions. This is true for most real cases except forabrupt changes in the symmetry axis in the area of complex faulting. However, some amount of smoothening is generally applied to the physical properties for stable simulation; therefore, this is a reasonable assumption. With this assumption, we rewrite the equation for H and H_0 as:

$$H = \frac{\partial}{\partial x}\cos^2\theta_0 b \frac{\partial}{\partial x} + \frac{\partial}{\partial z}\sin^2\theta_0 b \frac{\partial}{\partial z} - \left(\frac{\partial}{\partial x}\frac{\sin 2\theta_0}{2} b \frac{\partial}{\partial z} + \frac{\partial}{\partial z}\frac{\sin 2\theta_0}{2} b \frac{\partial}{\partial x}\right)$$

$$H_0 = \frac{\partial}{\partial x}\sin^2\theta_0 b \frac{\partial}{\partial x} + \frac{\partial}{\partial z}\cos^2\theta_0 b \frac{\partial}{\partial z} + \left(\frac{\partial}{\partial x}\frac{\sin 2\theta_0}{2} b \frac{\partial}{\partial z} + \frac{\partial}{\partial z}\frac{\sin 2\theta_0}{2} b \frac{\partial}{\partial x}\right)$$

The above equations can be represented in the form of the below operations;

$$H = \nabla \cdot \Theta \nabla \qquad (4)$$

$$H_0 = \nabla \cdot \Theta_0 \nabla \qquad (5)$$

with Θ and Θ_0 are the tensors.

We solved equations (4 and 5) using the support operator method [Shashkov, 1996]. It involves constructing conservative finite-difference schemes for solving differential equations. The method utilizes the **DIV** (divergence) and **GRAD** (gradient) operators to express the original differential equations. It also requires choosing specific locations on the grid for scalar and vector functions and discretizing the chosen primary operator. The discretization of the remaining operator is referred as the derived operator. By following these principles, the support operator method enables formulations of accurate and conservative finite difference schemes.

Numerical Results

In this section, we present the results of numerical simulation for TTI media using the developed algorithm. Thesimulation considered a physical domain measuring 3250 meters by 3250 meters, and the model is discretized using25-meter grid intervals. The velocity of the medium is 4000 *m/s*. To mimic an unbounded physical domain andminimize undesired reflections from the edges, absorbing layer boundary conditions were implemented on all four sides. Figure (1) shows the real and imaginary parts of the pressure wavefields at 15 Hz and 18 Hz frequencies. The wavefield exhibits elliptical shapes due to the anisotropicnature of the medium, and it is rotated at 45-degree angle, which is the direction of the principle axis of anisotropy. Subsequently, the frequency-domain results are transformed into the time domain using the Fourier transform to evaluate the accuracy of the response. For the time domain calculation, a Ricker wavelet with a dominant frequency of 10 Hz is taken as the source wavelet. Figure (2) illustrates time-domain wavefronts at three timesteps as 0.15 s, 0.25 s, and 0.35 s. It displays the same elliptical shape with a tilted major axis. The analytical P and S wavefronts, represented by the magenta and green curves, respectively, were obtained using the analytical VTI relation proposed by Payton(1983).

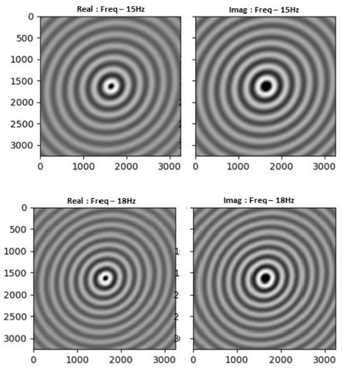

Figure 1: Real (left panel) and Imaginary part (right panel)of pressure wavefield in the frequency domain for a homogeneous TTI medium at 15 and 18 Hz frequencies

Initially, we computed the analytical wavefront for a

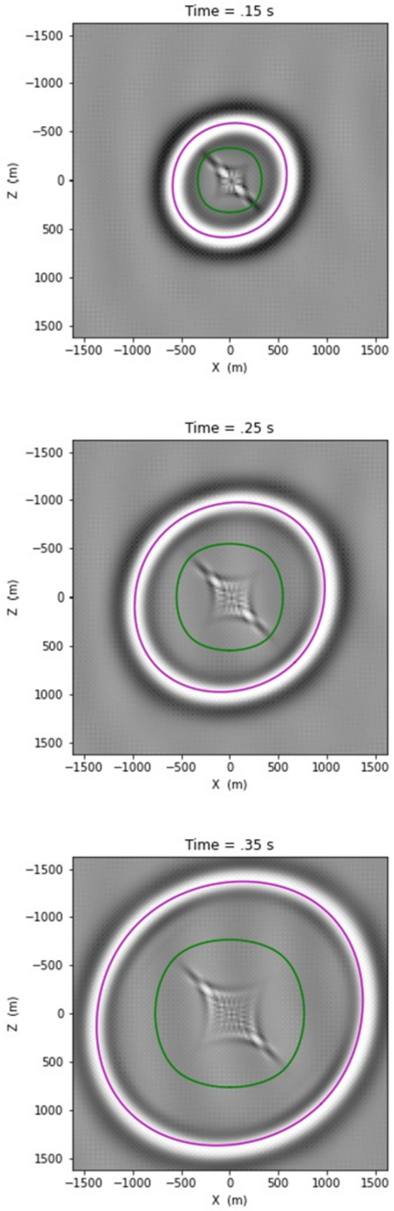

Figure 2: Pressure wavefield in the time domain for a homogeneous TTI medium at $t = 0.15s$, $t = 0.25s$ and$t = 0.35s$ respectively.

Support-operator method based frequency-domain acoustic-wave modeling in a transversely isotropic medium with tilted symmetry

VTI medium, and subsequently, we applied a 45-degree rotation to generate the corresponding TTI analytical wavefronts. This approach allowed us to accurately depict the propagation characteristics of both P and S waves in the TTI medium, leveraging the established VTI analytical results as a foundation. The tilt introduced by anisotropy influences wave propagation, necessitating accurate modeling for reliable subsurface material characterization in exploration seismology. Our simulations employing the support operator method capture these effects, facilitating anisotropic acoustic wave propagation modeling.

Conclusions

The proposed algorithm for modeling wave propagation in tilted transverse isotropic (TTI) media has proven to be both accurate and stable. By accurately simulating the behavior of wavefields, we have successfully captured the complex propagation patterns in such media. To validate our algorithm's accuracy, we conducted comparisons with analytical wavefronts, which further confirmed the reliability of the developed algorithm.

References

Tariq Alkhalifah. Acoustic approximations for processing in transversely isotropic media. *GEOPHYSICS*, 63 (2):623–631, 1998. doi: 10.1190/1.1444361. URL `https://doi.org/10.1190/1.1444361`.

Engelder, T., 1993, Stress Regimes in the Lithosphere. Princeton University Press, Princeton, NJ, 457pp.

Tariq Alkhalifah. An acoustic wave equation for orthorhombic anisotropy. *GEOPHYSICS*, 68(4):1169–1172, 2003. doi: 10.1190/1.1598109.

Jean-Pierre Berenger. A perfectly matched layer for the absorption of electromagnetic waves. *Journal of Computational Physics*, 114(2):185–200, 1994. doi: 10.1006/jcph.1994.1159.

Said Amiri Besheli, Milovan Urosevic, and Ruiping Li. The effect of seismic anisotropy on reservoir characterization. *SEG Technical Program Expanded Abstracts 2005*, 2005. doi: 10.1190/1.2144285.

José M. Carcione. Staggered mesh for the anisotropic and viscoelastic wave equation. *GEOPHYSICS*, 64(6): 1863–1866, 1999. doi: 10.1190/1.1444692.

Kai Gao, Shubin Fu, and Eric T. Chung. A high-order multiscale finite-element method for time-domain acoustic-wave modeling. *Journal of Computational Physics*, 360:120–136, 2018. doi: 10.1016/j.jcp.2018.01.032.

Bernhard Hustedt, Stéphane Operto, and Jean Virieux. Mixed-grid and staggered-grid finite-difference methods for frequency-domain acoustic wave modelling. *Geophysical Journal International*, 157(3):1269–1296, 2004. doi: 10.1111/j.1365-246x.2004.02289.x.

Heiner Igel, Peter Mora, and Bruno Riollet. Anisotropic wave propagation through finite-difference grids. *GEOPHYSICS*, 60(4):1203–1216, 1995. doi: 10.1190/1.1443849.

Zhiming Li, Bin Wang, Wenlong Xu, Gary Rodriguez, Chuck Mason, and Kwanjing Yoon. Impacts of seismic anisotropy and subsalt velocity updates in imaging, deepwater gulf of Mexico. *SEG Technical Program Expanded Abstracts 2008*, 2008. doi: 10.1190/1.3064098.

Ravish Mehra, Nikunj Raghuvanshi, Lauri Savioja, Ming C. Lin, and Dinesh Manocha. An efficient GPU-based time domain solver for the acoustic wave equation. *Applied Acoustics*, 73(2):83–94, 2012. doi: 10.1016/j.apacoust.2011.05.012.

Stéphane Operto, Jean Virieux, A. Ribodetti, and J. E. Anderson. Finite-difference frequency-domain mod- eling of viscoacoustic wave propagation in 2d tilted transversely isotropic (tti) media. *GEOPHYSICS*, 74 (5), 2009. doi: 10.1190/1.3157243.

R.-E. Plessix. A helmholtz iterative solver for 3d seismic- imaging problems. *GEOPHYSICS*, 72(5), 2007. doi: 10.1190/1.2738849.

Support-operator method based frequency-domain acoustic-wave modeling in a transversely isotropic medium with tilted symmetry

Mikhail Shashkov. *Conservative finite difference methods on general grids*. CRC Press, 1996.

Paul R. Williamson and R. Gerhard Pratt. A critical re- view of acoustic wave modeling procedures in 2.5 di- mensions. *GEOPHYSICS*, 60(2):591–595, 1995. doi: 10.1190/1.1443798.

Hongbo Zhou, Guanquan Zhang, and Robert Bloor. An anisotropic acoustic wave equation for model- ing and migration in 2d tti media. *SEG Techni- cal Program Expanded Abstracts 2006*, 2006. doi: 10.1190/1.2369913..