



Azimuthal Reflection Coefficients and Estimation of Anisotropic Parameters from Seismic Data

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Summary

In this paper, we study the dependence of P- and converted-wave reflection coefficients with incidence angle and azimuth for azimuthally anisotropic elastic media. Computing P- and converted-wave reflection coefficients using exact formulation, and inverting them using their asymptotic representations, we find that the P-wave reflection coefficient alone does not provide robust estimates of anisotropy parameters. If, on the other hand, the P-wave reflection is analyzed in conjunction with the converted-wave reflection, it is possible to get robust estimates of many useful anisotropic parameters. In the presence of fracture-induced anisotropy, these parameters can be directly linked with fracture orientation and fracture density. We, therefore, conclude that both P- and converted-wave data are necessary for estimating fracture parameters from surface seismic data.

Introduction

In an azimuthally anisotropic medium, the P- and converted-wave reflection coefficients vary with incidence angle and azimuth. These variations of P- and converted-wave reflection coefficients have been studied extensively to determine anisotropic parameters from seismic data.

In this paper, we outline the derivation of the reflection and transmission coefficients for an interface between two anisotropic solids, and also give their approximate forms for transversely isotropic media with a horizontal axis of symmetry (HTI), a property that is observable in nature when a set of oriented vertical fractures is present in an otherwise isotropic background. Using approximate coefficients, we outline a methodology for inverting observed reflection amplitudes for different anisotropic parameters. We then use our inversion methodology to extract reflection coefficient amplitudes and discuss the results of this inversion.

Exact reflection coefficients

Consider the propagation of a seismic wavefield in a depth-dependent anisotropic elastic medium. In the frequency-slowness domain, it can be shown that the displacement-stress vector field $\mathbf{b}$ satisfies the following equation (Fryer and Frazer, 1984)

$$\partial_z \mathbf{b} = i\omega \mathbf{A} \mathbf{b} + \mathbf{F}, \quad (1)$$

where $i = \sqrt{-1}$, ω is the circular frequency, $\partial_z = \partial/\partial z$ and

$$\mathbf{b} = [u_x, u_y, u_z, \tau_{xz}, \tau_{yz}, \tau_{zz}]^T, \quad (2)$$

is the vector containing the x-, y-, and z-components of the displacement vector $\mathbf{u}$ and the vertical component of the stress tensor $\boldsymbol{\tau}$. $\mathbf{A}$ in equation (1) is the 6X6 elastic system matrix whose components are functions of the elastic stiffness coefficients (c_{ij}), density (ρ), and x- and y components of the slowness p_x and p_y . Finally, $\mathbf{F}$ in equation (1) is the source term. Now, let the eigenvectors of the elastic system matrix $\mathbf{A}$ be given by the matrix $\mathbf{D}$, i.e.,

$$\mathbf{A} = \mathbf{D} \boldsymbol{\Lambda} \mathbf{D}^{-1} \quad (3)$$

so that $\boldsymbol{\Lambda}$ is the diagonal matrix of the eigenvalues. We also define a new vector $\mathbf{v}$, given as $\mathbf{b} = \mathbf{D} \mathbf{v}$. From equations (1) and (3), we can see that in a source-free region, the vector $\mathbf{v}$ satisfies the wave equation, and the individual components of $\mathbf{v}$ represent the up- and downgoing quasi-P (qP), quasi-S₁ (qS₁), and quasi-S₂ (qS₂) wavefield amplitudes, propagating with their respective vertical slowness values given by the eigenvalues in the diagonal elements of $\boldsymbol{\Lambda}$. The vector $\mathbf{v}$ is, therefore, called the wave vector. To compute reflection and transmission coefficients, we consider a discontinuity in elastic properties at a depth $z=z_1$. We also use the superscripts “-” and “+” to denote properties just above and below this discontinuity. Notice that the displacement-stress vector $\mathbf{b}$ is continuous across the discontinuity, i.e.,

$$\mathbf{b}(z_1^+) = \mathbf{b}(z_1^-). \quad (4)$$

But, because we have defined $\mathbf{b}=\mathbf{D}\mathbf{v}$, we can write from equation (4)

$$\mathbf{v}(z_1^+) = \mathbf{D}^{-1}(z_1^+) \mathbf{D}(z_1^-) \mathbf{v}(z_1^-) = \mathbf{Q}(z_1) \mathbf{v}(z_1^-), \quad (5)$$

where $\mathbf{Q}(z_1) = \mathbf{D}^{-1}(z_1^+) \mathbf{D}(z_1^-)$ is the wave propagator that propagates the wave vector across the discontinuity. The reflection and transmission coefficients are defined by the relationships between the up- and downgoing components of the wave vector fields above and below the interface. Using the above relationships, the downward and upward reflection and transmission coefficients $\mathbf{R}_d$, $\mathbf{T}_d$, $\mathbf{R}_u$, and $\mathbf{T}_u$ for the discontinuity are given as

$$\mathbf{R}_d = -\mathbf{Q}_{11}^{-1} \mathbf{Q}_{12}, \quad (6)$$

$$\mathbf{T}_d = \mathbf{Q}_{22} - \mathbf{Q}_{21} \mathbf{Q}_{11}^{-1} \mathbf{Q}_{12}, \quad (7)$$

$$\mathbf{R}_u = \mathbf{Q}_{21} \mathbf{Q}_{11}^{-1}, \quad (8)$$

and

$$\mathbf{T}_u = \mathbf{Q}_{11}^{-1}. \quad (9)$$

In equations (6)-(9), $\mathbf{Q}_{11}$, $\mathbf{Q}_{12}$, $\mathbf{Q}_{21}$, and $\mathbf{Q}_{22}$ are 3X3 submatrices of the 6X6 wave propagator matrix $\mathbf{Q}$. Also note that $\mathbf{R}_d$, $\mathbf{T}_d$, $\mathbf{R}_u$, and $\mathbf{T}_u$ are all 3X3, containing qP-qP, qP-qS₁, qP-qS₂, qS₁-qP, qS₁-qS₁, qS₁-qS₂, qS₂-qP, qS₂-qS₁, and qS₂-qS₂ reflection and transmission coefficients as components. Equations (6)-(9) give the reflection and transmission coefficients for an interface between two anisotropic solids. Exact details on deriving these coefficients and extending them to include fluid boundaries can be found in Fryer and Frazer (1984) and Mallick and Frazer (1991), respectively.

Approximate reflection coefficients

Although the reflection coefficients for a general anisotropic medium must be computed numerically, for media with higher orders of symmetry it is possible to obtain approximate expressions for these coefficients. Some of these approximate reflection coefficients for transversely isotropic media, with a vertical (VTI), horizontal (HTI), and tilted (TTI) axis of symmetry, can be found in Rüger (1996), Jilek (2000), and Sayers and Dean (2001) among others. These approximate coefficients are based on the first-order perturbation of the background isotropic medium and are valid for weakly anisotropic media. In a recent paper, Shaw and Sen (2004) showed that these approximate

reflection coefficients can be derived directly from the stationary phase evaluation of a Born integral.

In our feasibility study, we are interested in the HTI medium. Such a medium is usually observed in seismic exploration when a set of oriented vertical fractures are embedded in an isotropic background. Analysis of the reflection coefficients for such a medium can therefore be linked directly to the estimation of fracture parameters from seismic data. The elastic stiffness matrix $\mathbf{C}$ for a HTI medium is given by (e.g., see Rüger, 1996)

$$\mathbf{C} = \begin{pmatrix} c_{11} & c_{13} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{33} & c_{33} - 2c_{44} & 0 & 0 & 0 \\ c_{13} & c_{33} - 2c_{44} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{55} \end{pmatrix} \quad (10)$$

Following Rüger (1996), we now introduce the Thomsen parameters for HTI media as

$$\alpha = \sqrt{\frac{c_{33}}{\rho}}, \beta = \sqrt{\frac{c_{44}}{\rho}}, \mu = \rho\beta^2, Z = \rho\alpha, \quad (11)$$

$$\varepsilon^{(V)} = \frac{c_{11} - c_{33}}{2c_{33}}, \gamma = \frac{c_{44} - c_{66}}{2c_{66}}, \quad (12)$$

and

$$\delta^{(V)} = \frac{(c_{13} + c_{55})^2 - (c_{33} - c_{55})^2}{2c_{33}(c_{33} - c_{55})}. \quad (13)$$

In equation (11), α and β are vertical P- and S-wave velocities, μ and Z are vertical rigidity modulus and impedance, and ρ is the density. Using the notations from equations (11)-(13), the qP-qP and qP-qS₁ reflection coefficients R_{pp} and R_{psv} as functions of incidence phase angle (θ) and azimuth (ϕ) are given respectively as

$$R_{pp}(\theta, \phi) \approx \frac{1}{2} \frac{\Delta Z}{Z} + (b_i + b_a \cos^2 \phi) \sin^2 \theta + (c_i + c_2 \sin^2 \phi \cos^2 \phi + c_4 \cos^4 \phi) \sin^2 \theta \tan^2 \theta, \quad (14)$$

and

$$R_{psv}(\theta, \phi) \approx (f_i + f_a + g_a \sin^2 \phi) \sin \theta, \quad (15)$$



where

$$b_i = \frac{1}{2} \frac{\Delta \alpha}{\alpha} - 2 \left(\frac{\beta}{\alpha} \right)^2 \frac{\Delta \mu}{\mu}, \quad (16)$$

$$b_a = \frac{1}{2} \Delta \delta^{(v)} + 4 \left(\frac{\beta}{\alpha} \right)^2 \Delta \gamma, \quad (17)$$

$$c_i = \frac{1}{2} \frac{\Delta \alpha}{\alpha}, c_2 = \frac{1}{2} \Delta \varepsilon^{(v)}, c_4 = \frac{1}{2} \Delta \delta^{(v)}, \quad (18)$$

$$f_i = - \left\{ \frac{1}{2} \frac{\Delta \rho}{\rho} + \frac{\beta}{\alpha} \frac{\Delta \mu}{\mu} \right\} \quad (19)$$

$$f_a = \frac{1}{2 \left(1 + \frac{\beta}{\alpha} \right)} \Delta \delta^{(v)}, \quad (20)$$

and

$$g_a = \frac{1}{2 \left(1 + \frac{\beta}{\alpha} \right)} \Delta \delta^{(v)} + 2 \frac{\beta}{\alpha} \Delta \gamma. \quad (21)$$

All the material properties P in equations (14)-(21) are average properties and ΔP their contrasts across the interface. Note that we only use the first-order term in the converted-wave reflection coefficient formula because higher-order terms are unstable for parameter inversion (Jílek, 2001). We also assume here that the symmetry directions in both HTI media are the same.

Inversion of reflection amplitudes

From equation (14), the P-wave reflection amplitudes at azimuthal angles φ and for different incidence phase angles θ give the AVO intercept $A = 0.5(\Delta Z/Z)$, the AVO gradient B , and the higher-order AVO term C for a given azimuth as

$$B(\varphi) = b_i + b_a \cos^2 \varphi, \quad (22)$$

and

$$C(\varphi) = c_i + c_2 \sin^2 \varphi \cos^2 \varphi + c_4 \cos^4 \varphi. \quad (23)$$

We can, therefore, extract b_i , b_a , c_i , c_2 , and c_4 from $B(\varphi)$ and $C(\varphi)$. Once the AVO intercept A , and the parameters b_i , b_a , c_i , c_2 , and c_4 are estimated, and assuming the background S-to-P-wave velocity ratio β/α is known, it

is straightforward to use equations (16)-(18) to estimate the density contrast $\Delta \rho/\rho$, shear modulus contrast $\Delta \mu/\mu$, and the anisotropic parameters $\Delta \varepsilon^{(v)}$, $\Delta \delta^{(v)}$, and $\Delta \gamma$. In addition, notice that for fracture-induced anisotropy, $\Delta \varepsilon^{(v)}$ is nearly zero for wet (liquid-filled) fractures and greater than 0 for dry (gas-filled) fractures (for details, see Schoenberg and Douma, 1988). Consequently, $\Delta \varepsilon^{(v)}/\Delta \delta^{(v)}$ is the fluid factor attribute that is close to zero for wet fractures and non-zero for dry fractures. Now, the converted-wave reflection coefficient at a fixed azimuth φ , and its variation with the incidence phase angle θ gives the converted-wave reflection gradient $G(\varphi)$, and from equation (15), we write

$$G(\varphi) = (f_i + f_a) + g_a \sin^2 \varphi. \quad (24)$$

From the estimates of G for different azimuthal angles, we can get $(f_i + f_a)$ and g_a . Also note that from the P-waves, we can get b_i and b_a . We can then use equations (17) and (21), and β/α to estimate $\Delta \delta^{(v)}$ and $\Delta \gamma$. Finally, we use equations (19) and (20) to estimate the density contrast $\Delta \rho/\rho$ from $(f_i + f_a)$, $\Delta \delta^{(v)}$, β/α , and $\Delta \mu/\mu$.

Examples

For our feasibility studies, we use the wet and dry crack models given by Rüger (1996). For both models, the top layer was set isotropic with P-wave velocity (α): 3670 m/s, S-wave velocity (β): 2000 m/s and density (ρ): 2.41 g/cm³. The values used for the bottom layer in the wet and dry crack models are shown in Table 1.

For both models, we compute P- and converted-wave reflection coefficients using the exact and approximate formula. Figures 1 and 2 show the results of this computation along the directions perpendicular and parallel to the cracks.

Finally, we invert the exact reflection coefficient amplitudes using the P-wave only and combined P- and

Table 1: Details of the wet and dry crack models.

Parameter	Wet Crack	Dry Crack
α	4498 m/s	4388 m/s
β	2530 m/s	2530 m/s
ρ	2.8 g/cm ³	2.8 g/cm ³
Crack density	0.077	0.076
γ	0.0868	0.0868
$\varepsilon^{(v)}$	0	-0.1442
$\delta^{(v)}$	-0.0876	-0.1528

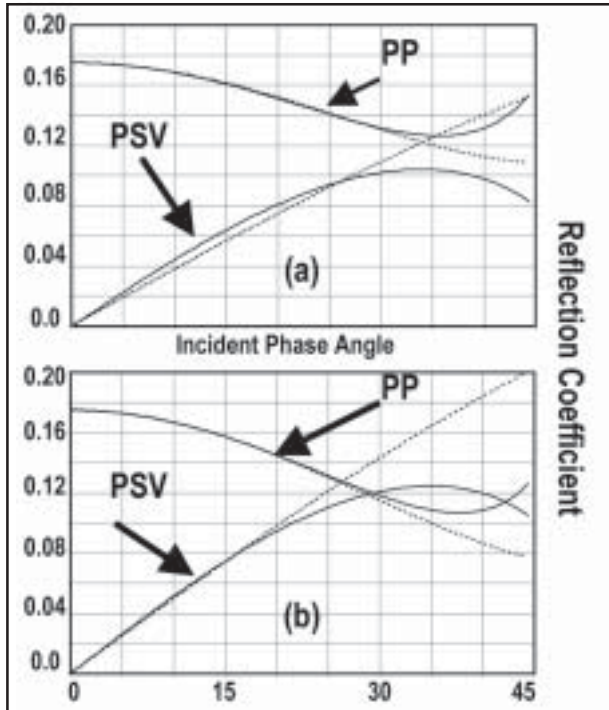


Fig. 1: Exact (solid line) and approximate (dotted line) reflection coefficients for the wet crack model at (a) perpendicular to, and (b) parallel to the cracks.

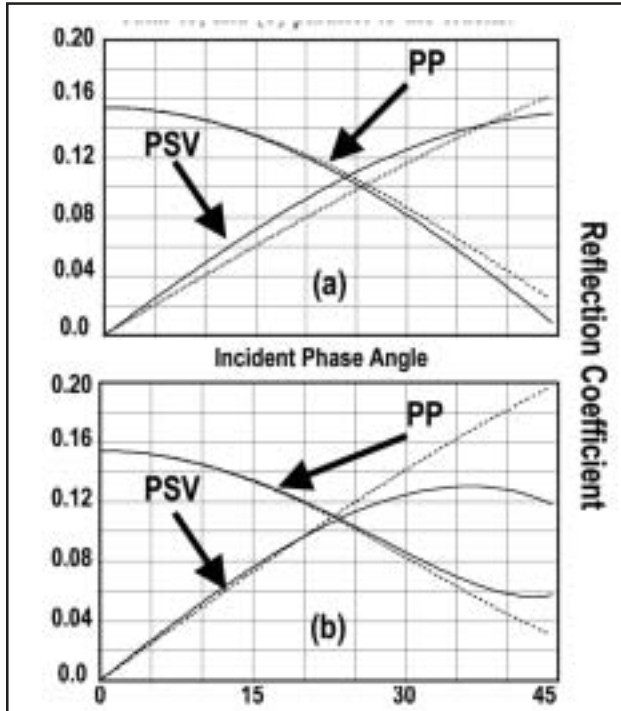


Fig. 2: Same as Figure 1, but for the dry crack model. Finally, we invert the exact reflection coefficient amplitudes using the P-wave only and combined P- and converted-wave analysis described as above. The results are shown in Table 2 and Table 3, respectively.

Table 2: Inversion of wet (liquid-filled) crack model.

Parameter	P- only	P and P-SV	True value
$\Delta\mu/\mu$	0.6376	0.6376	0.6001
$\Delta\rho/\rho$	-0.0357	0.1385	0.1497
$\Delta\gamma$	0.0373	0.0962	0.0867
$\Delta\varepsilon^{(v)}$	-0.3243		0
$\Delta\delta^{(v)}$	0.0092	-0.0839	-0.0876
Fluid factor	-0.0285		0

Table 3: Inversion of dry (gas-filled) crack model.

Parameter	P- only	P and P-SV	True value
$\Delta\mu/\mu$	0.6049	0.6049	0.6001
$\Delta\rho/\rho$	-0.0357	0.1407	0.1497
$\Delta\gamma$	0.1139	0.0956	0.0867
$\Delta\varepsilon^{(v)}$	-0.3773		-0.1442
$\Delta\delta^{(v)}$	-0.2475	-0.1553	-0.1528
Fluid factor	0.6562		0.9437

converted-wave analysis described as above. The results are shown in Table 2 and Table 3, respectively.

Discussion and conclusion

Figures 1 and 2 show that the approximate reflection coefficients match their exact counterparts out to incidence angles of 20-30°. Inversion results shown in Tables 2 and 3, on the other hand, demonstrate that robust estimates of anisotropic parameters are possible only when both P- and converted-wave data are used. Note that the P-wave only analysis relies on the higher-order AVO term in estimating the anisotropic parameters. Estimation of the higher order term requires reflection coefficients for angles beyond 35°, and from Figures 1 and 2 approximate coefficients at those angles do not match the exact coefficients very well. Therefore, it is not surprising that the P-wave analysis, requiring inversion to high angles, does not provide robust estimates. As P-wave data are less expensive to acquire than the multicomponent data, when cost becomes a factor, azimuthal variation of P-wave reflection amplitudes is used to find the fracture orientation either from the AVO gradient (Rüger, 1996) or from the fractogram analysis of stacked far-offset amplitude envelopes (Mallick et al., 1998). In addition, the ratio of the difference between the maximum and minimum azimuthal amplitude and the mean amplitude or the ratio of the gradient variation and the mean gradient is also used as a measure of the qualitative fracture density. As AVO gradients are more sensitive to noise than amplitude stacks, fractogram analysis is likely to be more stable than the AVO gradient analysis, and fracture orientation and qualitative



fracture density are the only parameters that are likely to be estimated with reliability from the P-wave data alone. The difference in stability of the two P-wave fracture analysis will be discussed during the presentation.

Unlike P-wave only analysis, P- and converted-wave analysis does not use the higher-order AVO terms. It only uses the 2nd order AVO term that dominates the values of reflection coefficients at incidence phase angles less than 25-30°. As approximate coefficients match reasonably with the exact coefficients at these angles, parameter estimates using a combined analysis are more robust. Also notice that because $\Delta\epsilon^{(V)}$ and fluid factor are obtained from the P-wave analysis, they are therefore not shown in the P- and converted-wave analysis results in Tables 1 and 2.

We conclude that multicomponent seismic data and a combined P- and converted-wave analysis are required for robust estimation of anisotropy and fracture parameters from seismic data.

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