TUTORIAL

In this session, we invite authors to submit articles which illustrate important concepts relevant to exploration geophysics.

WHY ZERO-PHASE FILTER SHOULD NOT BE APPLIED PRIOR TO DECONVOLUTION?

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When the seismic wavelet is not known, the seismic data are deconvolved using one-sided deconvolution which is based on an assumption that the seismic wavelet is of minimum phase. What harm may come to deconvolution if the data are acquired with a non-minimum phase wavelet or rendered non-minimum phase during pre processing as, for instance, using a zero-phase band pass filter? We answer this question in this tutorial and illustrate the requirement for the input data to be of minimum phase for effective deconvolution using a numerical and field example.

Let us first review different definitions of a minimum phase time series, also called a minimum-delay time series. As applied to seismic data, it could be a time series of a seismic trace, a source wavelet, or a response of a linear filter. A reader already knowledgeable about these definitions may like to proceed directly to examples.

Let,

$$x_t = \{ x_{-1}, x_0, x_1, \ldots\ldots x_n, \ldots\ldots \} \tag{1}$$

be a discrete time series obtained by sampling a function x(t) at a time sampling interval Δ t. For simplicity, we shall restrict discussion to only real functions x(t). We define a Z- transform of x_t as

$$X(z) = \sum_{-\infty}^{\infty} x_k Z^k \tag{2}$$

where

$$Z = \exp(i\,\omega\,\Delta t), \quad i = \sqrt{-1} \tag{3}$$

As an example, if we have a time series w_t,

$$w_t = \{ 0, \ldots\ldots a, \underset{\uparrow\, t=0}{1}, b, 0, 0 \ldots\ldots \} \tag{4}$$

its Z-transform is given by

$$W(Z) = a / Z + 1 + b\,Z \tag{5}$$

Z is called as a unit-delay operator since multiplying the expression (2) by Z generates another time series which is delayed by one sample with respect to (1). For instance, multiplying (5) by Z, gives

$$Z\,W(Z) = a + Z + b\,Z^2 \tag{6}$$

which defines a time-shifted series w_{t+1} as,

$$w_{t+1} = \{0, \ldots\ldots, 0, \underset{\uparrow\, t=0}{a}, 1, b, 0, \ldots\ldots\} \tag{7}$$

Note that the discrete Fourier transform, X(ω), of the time series (1), is obtained simply by substituting (3) into (2).

$$X(\omega) = \sum_{-\infty}^{\infty} x_k \exp(i\,\omega\,k\,\Delta t) \tag{8}$$

for $\quad -Ny \le \omega \le Ny$,

where $Ny = \pi / \Delta t$

is the Nyquist frequency (angular).

In this sense, the definition (2) *for Z-transform X(Z) has a useful characteristic that one can compute / examine the discrete F.T. of x_t in frequency domain by substituting the complex value of Z as in (8) and also view the underlying time series x_t as the coefficients of different powers of Z.*

Z-transform has some other useful features:

Convolution in the time domain corresponds to multiplication in the z domain, that is, if X(z), W(z), and R(z) are the Z-transforms of the sequences x_t, w_t, and r_t, respectively, then the convolution in time domain,

$$x_t = w_t * r_t \tag{9}$$

becomes a multiplication in the Z-domain,

$$X(z) = W(z)\,R(z) \tag{10}$$

A sequence for which

$x_t = 0$ for $t < 0$

is called a casual sequence. Another term for such a sequence is a "realizable" sequence. The word realizable comes from the fact that for any real time system (as different from a system designed numerically on a computer), the response of the system cannot come before the input; that is, the impulse response has to vanish for t<0. Conversely, one cannot design in real life a system for which $x_t \# 0$ for t<0, i.e. you cannot change the past (Philosophical, isn't it?). An example of a non-realizable filter is the zero-phase band pass filter. If it is of finite length, then one can make it "realizable", by delaying the response by half the length of the filter which introduces a similar delay in the output.

Further, if

$$\sum |x_n|^2 < \infty \tag{11}$$

i.e., *if a causal sequence x_n has a finite "energy", the sequence is called a realizable stable sequence.*

Consider a realizable sequence of a finite length,

$$x_t = \{x_0, x_1, x_2, \ldots\ldots\ldots, x_N\} \tag{12}$$

with a Z-transform,

$$X(Z) = x_0 + x_1 Z + x_2 Z^2 + \ldots\ldots\ldots\ldots x_N Z^N$$

$$= x_N (Z-z_1)(Z-z_2)\ldots\ldots\ldots\ldots(Z-z_N) \tag{13}$$

where $z_1, z_2, \ldots\ldots z_N$ are the roots of the polynomial X(Z).

From the property (10), the multiple products (13) in Z-domain represent a cascaded convolution in time domain. That is, the sequence x_t can be written as

$$x_t = x_N(-z_1, 1) * (-z_2, 1) * \ldots\ldots\ldots\ldots * (-z_N, 1) \tag{14}$$

A couplet (c_0, c_1) is said to be *minimum-delay* if $|c_0| \geq |c_1|$, i.e., if the energy is loaded in the front. A couplet (c_0, c_1) is said to be *maximum-delay* if $|c_1| \geq |c_0|$. In case $c_1 = c_0$, the couplet can be regarded as either minimum-delay or as a maximum-delay.

Definition of a Minimum Phase Sequence:

A sequence x_t is called a Minimum Phase sequence or minimum-delay sequence if all the zeroes of polynomial X(Z) lie outside the unit circle in the complex z-plane.
i.e.

$$|z_i| > 1, \text{ for } i = 1, 2, \ldots\ldots N.$$

Conversely, if all the roots z_i in (14) lie inside the unit circle, the sequence is called a maximum- delay sequence. If some roots are outside the unit circle and some inside, the sequence is called a mixed-phase sequence. *An important significance of the minimum phase property of a sequence x_n is that such a sequence has a one-sided inverse of finite energy*. This is simple to see as follows.

The inverse of a finite discrete sequence $\{x_0, x_1, \ldots\ldots\ldots x_n\}$ is defined by the equation

$$x_t * (x^{-1})_t = \delta_{t,0} \tag{15}$$

where,

$$(x^{-1})_t = \{\ldots\ldots, \underset{\substack{\uparrow \\ t=0}}{,x_{-1}^{-1}, x_0^{-1}}, x_1^{-1}, x_2^{-1} \ldots\ldots\ldots\}$$

is , in general, a two-sided time series , and

$$\delta_{t,0} = \{0, \ldots\ldots\ldots, 0, 1, 0, \ldots\ldots\ldots\ldots\}$$

is a spike at time t = 0

In Z-domain, equation (15) becomes,

$$X(Z)\, X^{-1}(Z) = 1 \tag{16}$$

Or,

$$X^{-1}(Z) = 1 / X(Z) = \frac{1}{X_N (Z-z_1) \ldots\ldots\ldots (Z-z_N)} \tag{17}$$

If all the roots of X(Z) are outside the unit circle,

i.e. if $|z_i| > 1$, for i =1, N,

then

$X(Z) \# 0$ for $|Z| \leq 1$.

i.e. $X^{-1}(Z)$ is analytic for $|Z| \leq 1$. It is this analyticity property of $X^{-1}(Z)$ which guarantees us that it can be expanded in a convergent power series around Z=0:

$$X^{-1}(Z) = \sum_{k=0}^{\infty} (x^{-1})_k Z^k, \text{ for } |Z| \leq 1, \tag{18}$$

with the property that

$$\sum |(x^{-1})_k|^2 < \infty \tag{19}$$

This proves the existence of a one-sided sequence $(x^{-1})_k$ of finite energy, which when convolved with x_k would give a spike. This existence criterion is not just of academic interest; we shall shortly see the relevance of this property of a minimum phase sequence for seismic deconvolution.

Let us note a few more important definitions and properties associated with minimum phase sequences.
For the sequence x_t as in (1), let us define a time reversed sequence

$$\bar{x}_t = \{ \ldots, x_1, \underset{\substack{\uparrow \\ t=0}}{x_0}, x_{-1}, x_{-2}, \ldots\ldots\ldots\} \tag{20}$$

For real valued x_t , the Z-transform of (20) is given by,

$$X^*(Z) = X(1/Z) \qquad (21)$$

which is just a complex conjugate function of X(Z).

Autocorrelation of x_t defined by

$$A(t) = \sum_{-\infty}^{\infty} x_k \, x_{t+k} \qquad (22)$$

can also be expressed as convolution of x_t with $\bar{x}_t$ (23)

$$A(t) = x_t * \bar{x}_t$$

In Z-domain, therefore, we have

$$A(z) = X(z)\,X^*(z) = X(Z)\,X(1/Z) \qquad (24)$$

Now, if X(Z) is a polynomial of minimum phase, then its roots $z_1, z_2, \ldots\ldots z_N$ defined in (13) all lie outside the unit circle. Accordingly, X(1/Z) which is a power series in (1/Z) representing an anti-casual sequence is given by

$$X(1/Z) = X_N (1/Z - z_1)(1/Z - z_2)\ldots(1/Z - z_N) \qquad (25)$$

$$= (X_N / Z^N)(1 - z_1 Z)(1 - z_2 Z)\ldots\ldots(1 - z_N Z) \qquad (26)$$

Note that all the roots of X(1/Z) are inside the unit circle, i.e. it represents a non-realizable (anti-causal) maximum-delay wavelet. Multiplying X(1/Z) by Z^N renders the wavelet causal, but it still remains of maximum phase.

We thus see that a power spectrum A(Z) of any times series X(z) can be factorized as

$$A(Z) = X_{MIN}(Z)\,X_{MAX}(Z) \qquad (27)$$

where $X_{MIN}(Z)$ is the minimum-phase factor of A(Z) with all roots of A(Z) outside the unit circle and $X_{MAX}(Z)$ is the maximum phase component with remaining roots of A(Z) lying inside the unit circle. Thus, given the auto-correlations $\{a_{-N}, a_{-N+1}, \ldots\ldots a_0, a_1, \quad, a_N\}$, to construct the minimum phase wavelet associated with it, we first construct the Z-transform of the spectrum:

$$A(Z) = \sum_{-N}^{+N} a_k Z^k \qquad (28)$$

We then make it causal by multiplying it by Z^N, which gives a polynomial of order (2N + 1) and obtain all the (2N + 1) complex roots. We then retain only those roots which lie outside the unit circle and construct the resulting polynomial. That gives us the minimum phase equivalent wavelet. We shall illustrate this technique by a numerical example later.

In frequency domain, the discrete F.T. (8) of x_t can be written as

$$X(\omega) = |X(\omega)| \exp\{i\Phi(\omega)\} \qquad (29)$$

where,

$$|X(\omega)| = |P(\omega)|^{1/2}$$

P(ω) is the power spectrum which is also the Fourier Transform of the Auto-correlation A(t) defined in (23).. Although we shall not prove it here, we state that if x_t is of minimum phase, then log $|X(\omega)|$ and phase Φ (ω) form a Hilbert Transform pair. i.e,

$$\Phi(\omega) = (1/\pi\omega) * \mathrm{Log}\,|X(\omega)| \qquad (30)$$

where * indicates convolution.
Thus, given the power-spectrum P(ω) we can construct the minimum phase Φ (ω). Note that *the property of minimum phase is not just a property of the phase function Φ(ω), but a property associated with a pair of functions - the power spectrum and the phase function. In other words, it is a property of the entire time series, in which the phase and the amplitude are intimately coupled and derivable from each other.* As a consequence, if we alter only the amplitude spectrum of a minimum phase time series without affecting its phase spectrum, say, for example, by applying a band-pass zero phase filter, then the resulting time series is no longer of minimum phase. This, as we shall see, has tremendous significance for seismic deconvolution.

Relevance of minimum phase property for seismic deconvolution :

In seismic context, we have a trace x_t modeled as

$$x_t = w_t * r_t$$

where r_t is the earth's reflecting series and w_t is the effective seismic wavelet (i.e. source wavelet convolved with the multiple response). When the wavelet w_t is known, one can always design a two sided filter f_k such that

$$w_t * \{f_{-2}, f_{-1}, f_0, f_1, \ldots\ldots\ldots\} = \delta_{t,0} \qquad (31)$$

When the filter is of a finite length, equation (31) has to be solved in a least square sense,

i.e., we need to find f_k (k = -N,.........N) which would minimize the error,

$$\sum [\delta_{t,0} - \sum f_k w_{t-k}]^2$$

with the standard Wiener filter solution:

$$\begin{pmatrix} a_0 & a_1 & & a_{2N+1} \\ a_1 & a_0 & & \\ & & & a_N \\ & & & a_{N-1} \\ a_{2N+1} & & & a_0 \end{pmatrix} \begin{pmatrix} f_{-N} \\ f_{-N+1} \\ f_0 \\ f_1 \\ f_N \end{pmatrix} = \begin{pmatrix} w_N \\ w_{N-1} \\ w_0 \\ 0 \\ 0 \end{pmatrix} \qquad (32)$$

Here, a_n are the auto-correlations of the wavelet and, when reflectivity is white, can be equated, for practical purpose, with the auto-correlations of seismic trace x_t .

The problem comes when the wavelet w_t is not known. In that case one searches for a one-sided filter $\{ f_0, f_1, \ldots \}$, i.e. $f_k = 0$, for k<0, such that

$$w_t * \{ 0,0, \quad ,f_0, f_1, \ldots f_N \} \quad = \quad \delta_{t,0} \tag{33}$$

How do we know that such a realizable stable sequence exists in the first place? It is here that we invoke the minimum phase assumption regarding the seismic wavelet. From the deliberations leading to equation(19), we know that if w_t is of minimum phase i.e. if all roots of W(Z) lie outside unit circle, then there exists $f_k = w_k^{-1}$ such that equation (33) is satisfied with $\sum |f_k|^2 < \infty$.

In that case, we can set $f_k = 0$ for k<0 in (32) and we get,

$$\begin{pmatrix} a_0 & a_1 & & a_N \\ a_1 & a_0 & & \\ & & & \\ & & & a_0 \end{pmatrix} \begin{pmatrix} f_0 \\ f_1 \\ \\ f_N \end{pmatrix} = \begin{pmatrix} w_0 \\ 0 \\ \\ 0 \end{pmatrix} \tag{34}$$

We now ask the question raised in the introduction. What happens if the seismic wavelet is not of minimum phase, but not knowing so, we blindly solve (34) and apply the filter to the input data? Let us see this with a specific example.

Consider a zero-phase wavelet

$$W_{ZP}(Z) = \qquad c / Z + 1 + c Z \tag{35}$$

To see, that it is indeed a zero phase wavelet, we only have to show that
$W_{ZP}(Z) = \{W_{ZP}(Z)\}^*$ where * represents complex-conjugate. In time domain, it represents a symmetric time series.

$$w_{zpt} = \{ 0,0, \ldots\ldots c, \underset{\substack{\uparrow \\ t=0}}{1}, c, 0, \ldots\ldots \} \tag{36}$$

The auto-correlations of w_{zpt} are

$$\left. \begin{aligned} a_0 &= 1 + 2c^2 \\ a_1 &= a_{-1} = 2c \\ a_2 &= a_{-2} = c^2 \end{aligned} \right\} \tag{37}$$

and $a_n = a_{-n} = 0$ for n> 2

In Z-domain,

$$A(Z) = W_{ZP}(Z) \quad W_{ZP}(1/Z)$$

$$= (c / Z + 1 + c Z)^2 \tag{38}$$

One can verify that expansion of (38) as $\sum_{k=-\infty}^{\infty} a_k z^k$

leads to coefficients a_0, a_1, a_2 precisely as in (37).

The expression (38) can also be rewritten as

$$A(Z) = (c^2 / Z^2) (Z - Z_-)^2 \; (Z - Z_+)^2 \tag{39}$$

where the two roots are given by

$$Z_{\pm} = - (1/2c) [1 \pm \operatorname{sqrt} (1 - 4 c^2)] \tag{40}$$

For simplicity, consider the case $4c^2 < 1$.i.e. c<1/2, so that both the roots Z_+ & Z_- are real with

$$Z_- > 1, \text{ and } Z_+ < 1 \tag{41}$$

Now, we can identify the minimum phase equivalent of the zero-phase wavelet (35) as given by the factors of A(Z) which have roots

$$W_{MIN}(Z) \sim (Z - Z_-)^2 \tag{42}$$

For proper normalization, we define

$$W_{MIN}(Z) = (-c / Z_-) \{ (Z - Z_-)^2 \} \tag{43}$$

so that we can express the spectrum (39) precisely as

$$A(Z) = W_{MIN} (Z) \, W_{MAX} (Z) \tag{44}$$

where

$$\begin{aligned} W_{MAX}(Z) &= -(cZ_- / Z^2) (Z - Z_+)^2 \\ &= W_{MIN} (1/Z) \end{aligned} \tag{45}$$

Using the property $Z_- \; Z_+ = 1$, one can verify that the spectrum of W_{MIN} (Z) defined in (43) is the same as A(Z) defined in (39) or (44).

In time domain, we have

$$w_{min\,t} = \{ 0, \ldots 0, \underset{\substack{\uparrow \\ t=0}}{-c z_-}, 2c, - c / z_-, 0, \ldots \} \tag{46}$$

If we take $c = \sqrt{2}/3$, $Z_- = -\sqrt{2}$, and the minimum phase equivalent wavelet becomes

$$w_{min\,t} = \{ 0,0 \ldots 0, \underset{\substack{\uparrow \\ t=0}}{0.6666}, 0.9428, 0.3333, 0,0 \ldots \} \tag{47}$$

The autocorrelations of the $w_{min\,t}$ and $w_{zp\,t}$ as given by (37) are

$$a_0 = \quad 1.4444, a_1 = 0.9428, a_2 = 0.2222 \tag{48}$$

$$a_3 = \quad a_4 = \ldots\ldots = 0$$

We now design a one-sided inverse filter of expression of (47). The exact inverse is given by $1/W_{MIN}(Z)$, which has an infinite number of terms when expanded in powers of Z. However, let us be content with a 3-terms approximation of the inverse filter which is obtained by substituting the values of $a_0, a_1, \ldots$ and w_0 into (34)
i.e.

$$\begin{pmatrix} 1.4444 & 0.9428 & 0.2222 \\ 0.9428 & 1.4444 & 0.9428 \\ 0.2222 & 0.9428 & 1.4444 \end{pmatrix} \begin{pmatrix} f_0 \\ f_1 \\ f_2 \end{pmatrix} = \begin{pmatrix} 0.6666 \\ 0 \\ 0 \end{pmatrix} \qquad (49)$$

with the solution

$$f_t = (1.03739, -0.99819, 0.49192) \qquad (50)$$

When the inverse filter is convolved with (47), we get a deconvolved wavelet, say, d_{min},

$$d_{min} = \{0, \ldots\ldots \underset{\underset{t=0}{\uparrow}}{0.69159}, 0.31258, -0.26735, 0.13105, 0.16397, 0..\} \qquad (51)$$

The least square error of deconvolution is given by

$$LSE = \sum_t \{\delta_{t,0} - (w_{min\,t} * f_t)\}^2 \qquad (52)$$

which computes to 0.308364.

The minimum phase wavelet (47) and the deconvolved output (51) are displayed in figure 1 (a) and (b), respectively.

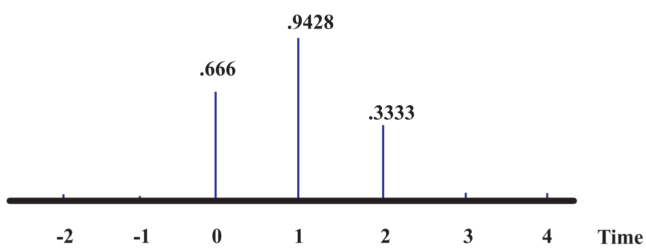

Fig.1(a). A minimum phase wavelet corresponding to equations (43) and (47).

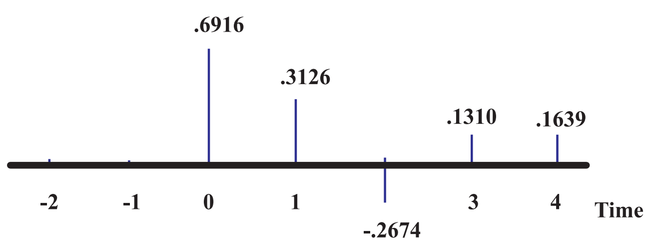

Fig. 1(b). Result of applying a 3-point spiking deconvolution on the input Fig 1(a) with Least Square Error=.308364

If now, *unmindful of the fact that a non-minimum phase wavelet does not have a one-sided inverse,* we try to find such an inverse filter for the zero-phase wavelet

$$w_{zpht} = \{0, 0, \ldots\ldots\ldots\ldots 0.4714, \underset{\underset{t=0}{\uparrow}}{1}, 0.4714, 0, \ldots\ldots\ldots\ldots\} \qquad (53)$$

the filter would again be the solution of the same equation (44) with the entry in the top row on the R.H.S corresponding to w_0 in (34) being replaced by 1.0. The corresponding filter is given by

$$\{0, 0, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 1.5561, \underset{\underset{t=0}{\uparrow}}{-1.4973}, 0.7379\} \qquad (54)$$

which when convolved with (53) gives

$$d_{zpht} = \{0, 0, \quad 0.7335, \underset{\underset{t=0}{\uparrow}}{0.5302}, -0.4159, .0320, .3478, 0\} \qquad (55)$$

with a least square error, LSE = 1.0537

The Zero-phase wavelet (53) and the result (55) of applying one-sided inverse filter (54) are shown in figure 2(a) and (b), respectively. *The comparison of figures 1 & 2 shows the deleterious effect of applying one-sided deconvolution on non-minimum phase input.*

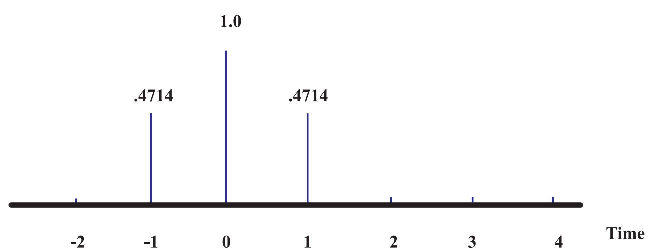

Fig. 2(a). A zero-phase wavelet corresponding to equations (35) and (53). By design, this wavelet and the minimum phase wavelet in Fig 1(a) have the same spectrum given by equations (44) and (48).

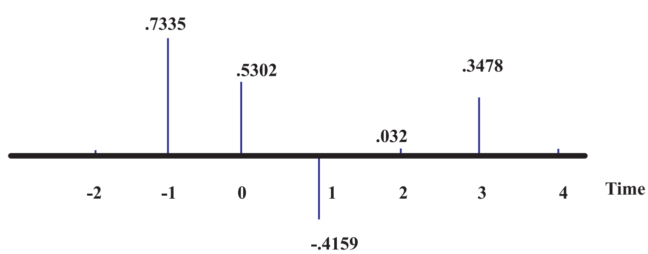

Fig. 2(b). Result of applying a 3-point spiking deconvolution on the input Fig 2(a), with Least Square Error=1.0537.

Effect of Zero-phase Filtering :

We get similar undesirable effect when a zero-phase filter is applied prior to spiking deconvolution on data which are minimum phase. In figure 3(a) we have shown an input with a spike at 200ms. A small random noise of 0.01% is added. Figure 3(b) shows the result of applying a zero-phase band-pass (b.p.) filter, (filter parameters are not very relevant for the discussion). The good thing about zero-phase filter is that since the phase is not changed, the location of the peak remains at zero time. The bad thing, in fact, totally unacceptable thing about zero-phase filtering is the result shown in figure 3(c) obtained by convolving this filtered wavelet with a one-sided spiking deconvolution filter.

The same exercise carried out with minimum-phase b.p. filter applied on a spike input is shown in figure 4(b). Clearly, the filtered output has a much lower resolution and a longer

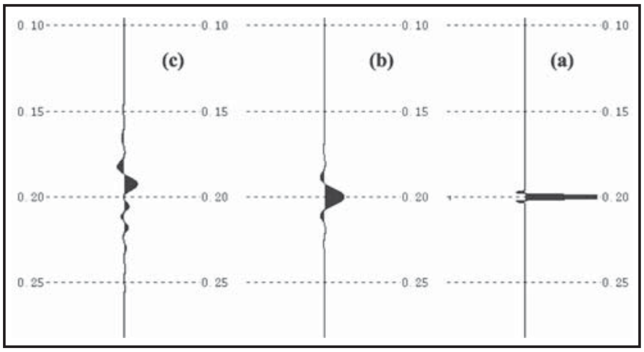

Fig. 3. An input with a spike (extreme right) at 200ms with random noise of 0.01% (b) The result (middle) of applying a zero-phase band-pass filter (c) Result of applying a one-sided spiking deconvolution to the filtered wavelet in (b).Compare the results with those in Fig. 4

length than in the output figure 3(b) with zero-phase band-pass filter. And this is the attraction of zero phase filtering. However, the final deconvolved result in figure 4(c) is a spike, as desired, much superior to the result in Fig 3 (c). *This illustrates unsuitability of non-minimum phase input - in this case, resulting from a zero phase band pass filter - for one-sided deconvolution of data.*

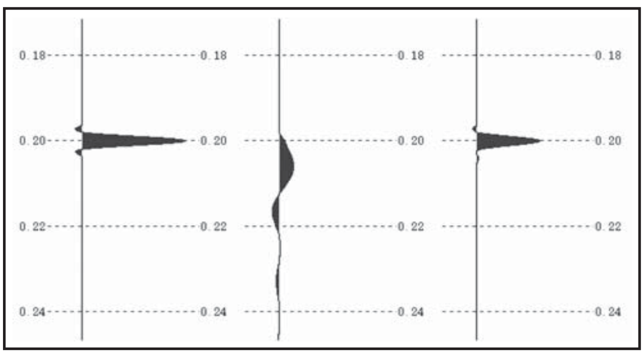

Fig. 4. An input with a spike at 200ms with a random noise of 0.01%. (b) The result of applying a minimum-phase band-pass filter. (c) Result of applying a one-sided spiking deconvolution to the filtered wavelet in (b).Compare the results with those in Fig. 3.

Finally, let us see the effect of making the same mistake of applying a zero-phase filter on a real data set. For illustration we have used data acquired with an array of 64 air-guns. The arrays are designed to obtain high primary-to-bubble ratio which attenuates the bubble effect at least in the vertical direction ensuring that the effective source signature would be close to minimum phase. (That is, if the source signature is decomposed into the polynomials W_{MIN} with all zeroes outside the unit circle and W_{MAX} with all zeroes inside the unit circle, most of the total energy of the source would be contained in W_{MIN}).

Figure (5) shows the effect of zero-phase filtering the data with filter parameters:

LC	= 4	Hz,	Slope	= 12 db/octave
HC	= 60	Hz,	Slope	= 72 db/octave.

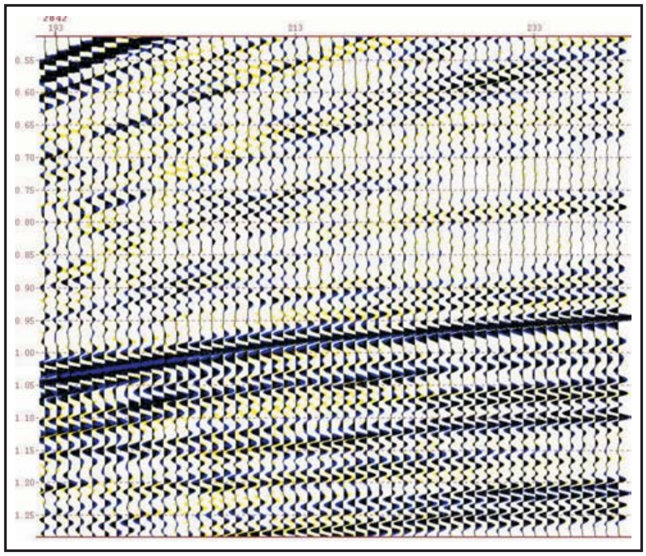

Fig. 5. The effect of zero-phase filtering a sample marine record with filter parameters (4 Hz, 12 db/oct, 60 Hz, 72 db/oct). The filtered output is shown in blue, while the input is shown in yellow. Black indicates the overlap between the input and the filtered output. Apart from the side-lobes associated with zero-phase b. p. filters, the positions of the peaks and troughs of the input are not altered.

The filtered output is shown in blue, while the input is shown in yellow. Black indicates the overlap between the input and the filtered output. As is evident form this figure, apart from the side-lobes associated with zero-phase b. p. filters, the positions of the peaks and troughs of the input are not altered. This is the nice aspect of zero-phase filters and it is for this reason that many processors like to use zero-phase filters. Figure (6), shows the effect of applying minimum phase b. p. filter with the same filter parameters as before. The input data, this time, is shown in blue, while the filtered output is shown in yellow. Again the black portion indicates the overlap between the input and the filtered output. As can be seen, the filtered output (yellow) is delayed with respect to the input (by about 8-10 msecs), and appears to have lesser resolution than the input.

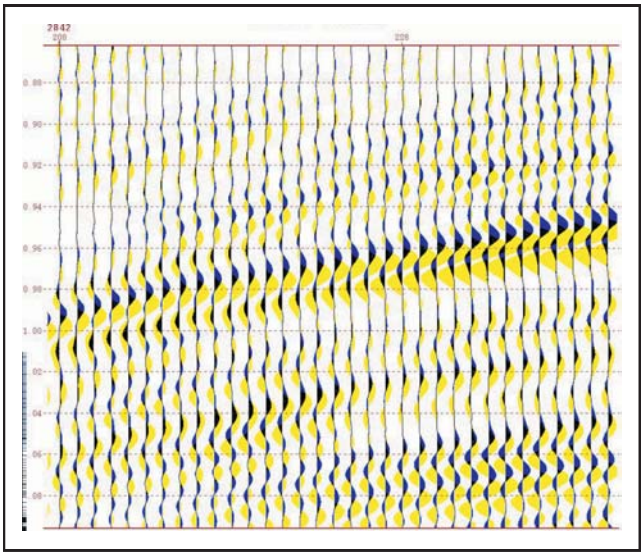

Fig. 6. The effect of applying minimum phase b.p. filter with the same filter parameters as for Fig. 5. The input data, this time, is shown in blue, while the filtered output is shown in yellow. The black portion indicates the overlap between the input and the filtered output. Observe that the minimum phase filtered output (yellow) is delayed with respect to the input (by about 8-10 msecs), and appears to have lesser resolution than the input. However, also see the Figure 8 to see the post-decon data.

Figure (7) shows the comparison between the minimum phase filtered output (in blue) and the zero-phase filtered output (in yellow). Note the time-shift of the former. So far so good!

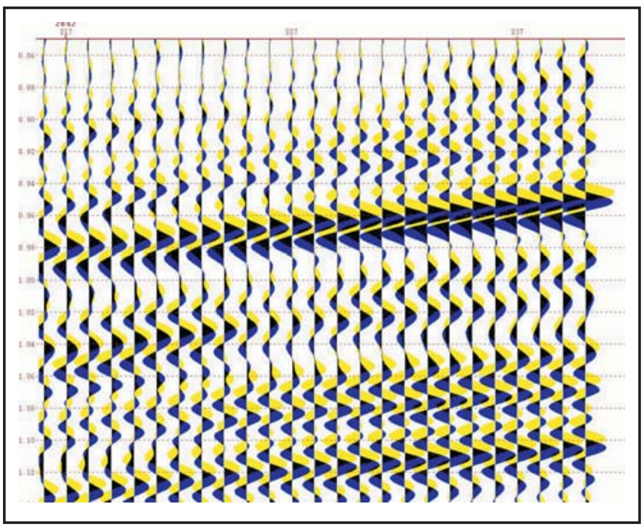

Fig. 7. Comparison between the minimum phase filtered output (in blue) and the zero-phase filtered output (in yellow). Note the time-shift of the former.

It would appear from the forgoing that one should use only zero-phase b. p. filter in preference to minimum phase b. p. filter as the former does not change the phase of the input. But wait! Let us compare the result of spiking deconvolutions applied on minimum-phase b. p. filters which is shown in figure 8(a), with the spike-decon result shown in figure 8(b) on the zero-phase b. p. filtered input. For comparision, the raw data which were input to the exercises in figure 8 (a) and (b) are shown in figure 8(c). As was the case with the numerical and synthetic examples shown earlier, the spiking deconvolution has resulted in a higher resolution for minimum phase b. p. filtered input than for the zero-phase b.p. filtered input. The stand-out of events is also better for the minimum-phase case than for the zero-phase case. The spectra of the input record, deconvolved data with zero and minimum phase b.p. filters applied prior to deconvolution input are displayed in figure 9(a) 9(b) and 9(c), respectively. The bandwidth is higher for (c) the minimum phase pre-decon filter than in (b) the zero-phase pre-decon filter.

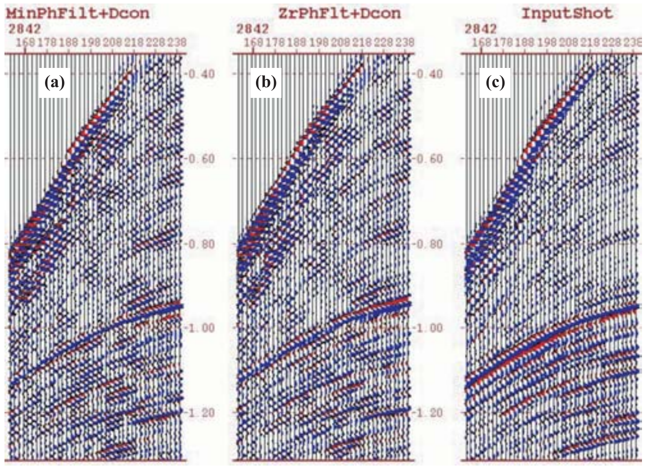

Fig. 8. (a) Result of applying minimum phase filter followed by a spiking filter. (b) Result of applying zero-phase filter followed by a spiking filter. (c) The raw data used for input for the processes in figures (a) & (b). Observe the higher resolution (and better stand-out) of events for the minimum phase pre-decon filter (a) as compared to that for the zero-phase case (b).

The reason for superior performance of deconvolution on minimum phase b.p. filter compared to that in zero-phase b.p. filter should be obvious by now. The zero-phase b.p. filters retains the input phase while altering the amplitude spectrum. Consequently, the minimum phase property of the input (to the extent that it was there in the data) is destroyed rendering the data unsuitable for spiking deconvolution.

These differences between using minimum-phase filter and zero-phase filter prior to decon appear small when viewed on the final stack data, but are of critical nature for pre-stack analysis of the data. Imagine someone carrying out AVO analysis for the strong event around 950 millisecs (for near offsets) on the data sets using a zero-phase filter prior to decon displayed in figure 9(b). Because of mixing of the primary event with decon effect of the side lobes, the amplitudes in the middle offsets would be distorted compared to those for a similar analysis with the data in figure 9(a).

We leave it to the reader to satisfy himself that the above deliberations also hold when a predictive deconvolution, which would try to retain a part of the source wavelet but suppress short-period multiples, is applied.

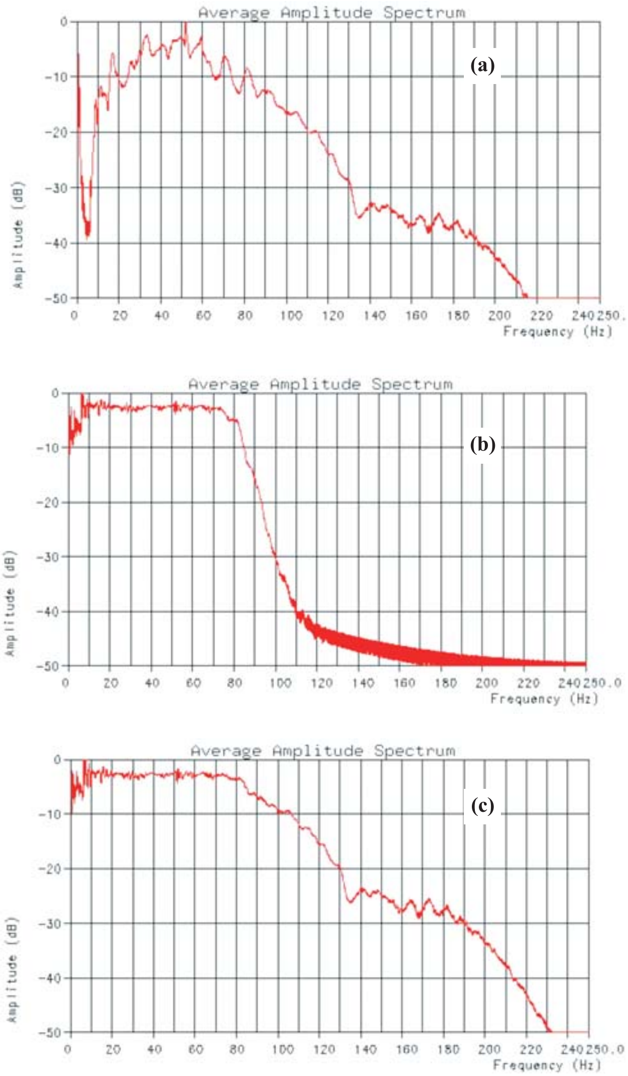

Fig. 9. The spectra of (a) the raw input (b) deconvolved data (with zero phase b.p. filter) and (c) with minimum phase b. p. filter.

Suggested readings

1. "Fundamentals of Geophysical Data Processing with Applications to Petroleum Prospecting", Claerbout Jon F, 1985, Blackwell Publications. One of the best text books for a person who lacks sufficient mathematical background of data processing, but wishes to acquire it. Chapter 2 on one-sided functions and 3 on spectral factorization (for construction of minimum phase wavelet) supplement this article.
2. "Predictive Decomposition of the Time Series with Application to Seismic Exploration", E.A. Robinson, Geophysics, Vol 32, No. 3, 1967, P 418-484. A pioneering paper in seismic deconvolution. A "must-read" for a geophysicist who wishes to grasp the foundation of seismic deconvolution and is not averse to its mathematics.
3. "Predictive decomposition: Theory & Practice" K.L. Peacock and Sven Trietal, Geophysics, VOl.34, No. 2., 1969 p. 155-169. This paper extends the work of E. A. Robinson to multiple suppression and illustrates it with examples.
4. "Least-squares Inverse Filtering and Wavelet Deconvolution" A.J. Berkhout, Gephysics, Vol 42, 7, 1977, P. 1369-1383. The paper describes wavelet deconvolution (same as two sided inverse filtering when noise is white) and illustrates its advantages over the one-sided inverse filtering (based on minimum-phase assumption) in terms of higher resolution of the deconvolved data.
5. "Minimum phase wavelet by ARMA factorization", C.H. Mehta, B. S. Goel, D. D. Bhatt & S. Radhakrishnan), IEEE trans. on Acoustic, Speech and Signal Processing. Vol.32, No.2 (USA), (Feb.1991). Construction of a minimum phase wavelet by factorization of the auto-correlations has a serious numerical limitation for long wavelets. This paper describes a practical and accurate method for minimum phase wavelet construction by factoring ARMA representation of the causal component of auto correlations z_1.