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Inversion of Schlumberger Vertical Electrical Sounding Data using Hybrid Monte Carlo Based Bayesian Neural Network Approach

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Summary

Interpretation of vertical electrical sounding (VES) data is crucial for groundwater exploration. The study area occupies crystalline hard rock of Pre-Cambrian hornblende and feldspathic gneisses, schists, granulites, quartzites, metabasics and pegmatities. We develop here a novel algorithm based on the Bayesian neural network (BNN) theory using the concept of Hybrid Monte Carlo (HMC)/ Markov Chain Monte Carlo (MCMC) simulation scheme for direct inversion of direct current (DC) VES measurements from the 30-locations around the Tenduli- Vengurla, Sindhudurg district, coastal Maharashtra, India. The inversion results suggest that the top layer is mostly comprised of laterites followed by mixture of clay/clayey sand and garnulites/granite as basement rocks. The water strikes weathered/semi-weathered layer of laterite/clayey sand within the depth of 10-15m from the surface. Two dimension inversion via the HMC-BNN method of three resistivity profiles data from the study region demarcate two potential groundwater reservoirs; one lying between Path-Tenduli and another in between Math and Zaraph. The inverted true electrical resistivity section against depth correlates well with available borehole lithology in the area. The results presented here are useful for interpreting the geological signatures like fractures, major joints and lineaments, which in turn, will be useful for identifying groundwater reservoirs and drainage pattern in the crystalline hard rock area.

Introduction

Groundwater is the major source that meets the drinking and domestic needs of any area of the world. Thus, its exploration and accessibility assumes a great significance to the scientists and society. In the crystalline hard rock groundwater generally occurs in the weathered basement, or regolith, and the fractured rock(Verma et al., 1980). The presence of weathered and fractured quartzites and granites, generally associated with weathered zones may enhance the chances of high yielding boreholes. Many of the communities in Sindhudurg district are located where there is almost complete dearth of sand and gravel aquifers, which are favorable for constructing high-yielding boreholes. In the overburden, the aquifers are predominantly clayey sand. Most of the water is, however, encountered at the interface between the regolith and the fractured bedrock. Furthermore near the coastal region adjacent to the Arabian Sea, groundwater level fluctuates in response to tidal variation. As a result there is always a possibility of saltwater intrusions in fresh water pockets in the coastal belt of Maharashtra. This leads to the pollution of groundwater (Song, et al, 2007). Hence, understanding the fluid percolation pattern near the sub-surface of costal area and mapping the intrusion of saline water through pores and geologically weak zones, such as faults and fractured zones is of considerable societal importance. Modeling and interpretation of Direct Current (DC) resistivity sounding in the region would play chief role in elucidating the above patterns.

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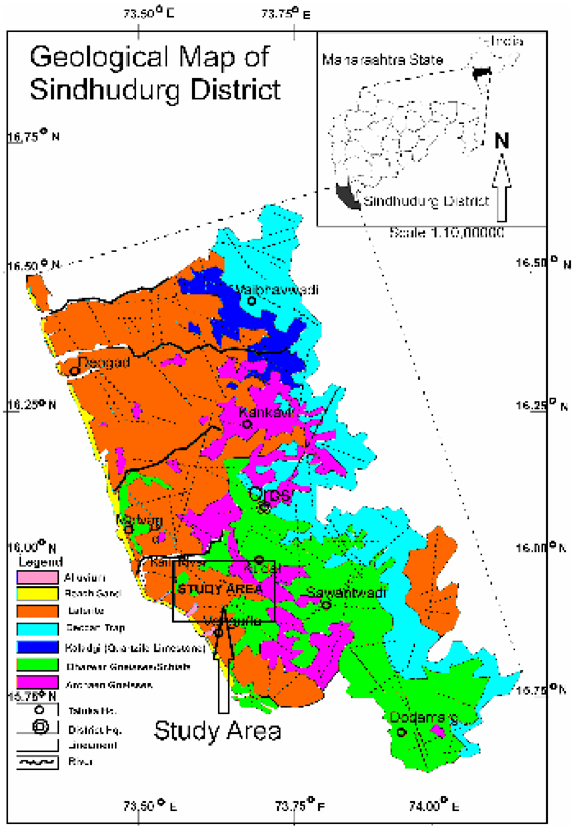

Figure 1: Geological map of the study area

Direct Current (DC) resistivity sounding method is one of the most popular methods that have been extensively applied for solving hydrological, geothermal, environmental and engineering problems (Zohdy, 1989; Maiti et al., 2011).

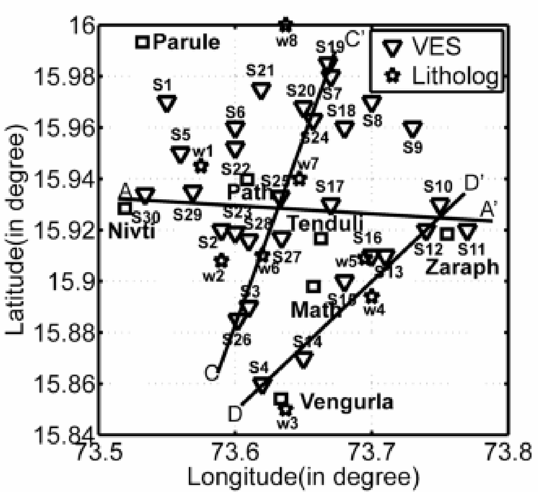

Figure 2: Location map of Schlumberger sounding

In the DC resistivity method, current is introduced directly into the ground through a pair of current electrodes and resulting voltage difference is measured between a pair of potential electrodes. The method provides the apparent resistivity distribution against depth. The depth of penetration of electrical signal is generally found to be approximately one-third of the distance between the electrode separations.

The relation between the observed "apparent resistivity" and model parameter ("true resistivity" and "layer thickness") is non-linear. Hence the estimation of true resistivity distribution against depth from apparent resistivity data essentially leads to solving the nonlinear inverse problem. Several attempts have been put forward to solve the resistivity inverse problems (Ghosh 1971; Zohdy, 1989). The existing methods suffer from common drawbacks such as these algorithms critically depend on the initial parameter chosen for it. The general class "Monte Carlo" e.g., genetic algorithm, simulated annealing is proven to be useful when a good staring model is unavailable (Rubinstein, 1981; Kirkpatrick 1983; Bhattacharya et al., 2003) for providing global solution. These methods are however computationally expensive and unfeasible for processing voluminous data.

Artificial Neural Network (ANN) has been become popular in geophysical community because the method can approximate any functional relationship between two domains with arbitrary accuracy even if the relationship is nondeterministic. The techniques have been widely applied to solve non-linear problems in almost all branches of geophysics (e.g., Van der Baan and Jutten, 2000; Poulton, 2001).

There are, however, several limitations in conventional neural network approaches also (see Bishop 1995; Maiti et al., 2011). One of the main problems is that the network is learned by maximizing a likelihood function of the parameters or equivalently minimizing an error function in order to minimize the objective function by adjusting networks parameters. Sometimes a regularization term with an error function is incorporated to prevent over-fitting. In that case, a complex model can well fit training data but it does not necessarily guarantee smaller errors in new data (see Bishop, 1995). This is because it does not take into account uncertainty in the estimation of network parameters (see Bishop 1995).

Recently developed a Bayesian neural network approach via a powerful Hybrid Monte Carlo approach (see Maiti et al., 2011) is used here for estimating true resistivity and layer thickness precisely from the grided apparent resistivity data (30 Schlumberger sounding profiles) collected over a of Tenduli-Vengurla region Maharashtra (Figure 1 &2). We also compare the new HMC-based BNN results with the results of conventional singular value decomposition (SVD) (see Maiti et al., 2011). Our results suggest that the HMC-based BNN technique is superior to conventional SVD-based techniques in a sense that the present technique takes care of the problem of uncertainty analysis in a natural way. Thus, besides introducing a novel HMC-based BNN approach for solving non-linear inverse problem, the analysis brought out precise distribution of true resistivity which is especially useful for groundwater exploration in crystalline hard rock area of Tenduli-Vengurla, Konkan region.

Theory and /or Method

For Schlumberger sounding, the relationship between the apparent resistivity (ρ_a) and the layer parameters (e.g. layer thickness, layer true resistivity) can be expressed by an integral equation considering an earth model consisting of homogeneous and isotropic layers. We write the equation following (Koefoed, 1970).

$$\rho_a(s) = s^2 \int_0^\infty T(\lambda) J_1(\lambda s) \lambda d\lambda \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Where, s is half the current electrode spacing (AB/2) in Schlumberger electrode configuration, J_1 denotes the first-order Bessel function of the first kind and λ denotes the integral variable. Following Koefoed (1970) we write recurrence relationship of the resistivity transform function, $T(\lambda)$ as,

$$T_i(\lambda) = \frac{T_{i+1}(\lambda) + \rho_i \tanh(\lambda h_i)}{[1 + T_{i+1}(\lambda)\tanh(\lambda h_i / \rho_i)]}, i = n-1 \ldots.1 \quad (2)$$

Where, n denotes the number of layers, ρ_i and h_i are the true resistivity and thickness of the i^{th} layer respectively.

Inversion scheme

The standard linearized inversion approach to solving the non-linear inverse problems in geophysics has been iterative. However, the inversion of the resistivity sounding data is an ill-posed problem. The reason is being that the contradictory information on model parameter cannot be assessed due to lack of information. Therefore, small changes in the data may lead large changes in the model. Successful optimization depends heavily on the choosing the correct initial model. The popular damped least-squares based solution can be written as (Menke, 1984).

$$\Delta m = (G^T G + \beta^2 I)^{-1} G^T \Delta d \quad \ldots\ldots\ldots(3)$$

Where Δm is the parameter correction vector; Δd is the data difference vector, G is the Jacobian matrix contains partials derivative of data with respect to the initial model parameters. I is the identity matrix, and the term β is called damping factor which is a scalar quantity actually controls both speed of convergence and solution. This solution is also known as Tikhonov regularization (Menke, 1984).

Bayesian neural network approach

In the conventional ANN approach, a network is trained using a finite data set (here, model /data pair), $s = (\{d_k, x_k\})^{N}{}_{k=1}$ by adjusting network parameters w (weight and biases) so as to minimize an error/misfit function such as

$$E_s = \frac{1}{2}\sum_k^N \left\{ x_k - o_k(d_k; w_k) \right\}^2 \quad \ldots\ldots\ldots\ldots(4)$$

The minimization of the error is based on repeated evaluation of the gradient of E_s using "back-propagation" (Bishop, 1995). Often, regularization is included to modify misfit function

$$E(w) = \mu E_S + \lambda E_R \quad \ldots\ldots\ldots\ldots\ldots\ldots(5)$$

Where, for example, $E_R = 0.5\sum_{i=1}^{R} w_i^2$, w is total number of weights and biases in the network, λ and μ, which control other parameters (synaptic weight and biases), are known as hyper parameters. It is important to note that forward functions used in node are nonlinear tan sigmoid in nature eases to solve the non trivial problem (Figure 3). In the traditional approach, the training of a network starts with a initial set of weights and biases and ended up with the single best set of weights and biases given the objective function is optimized.

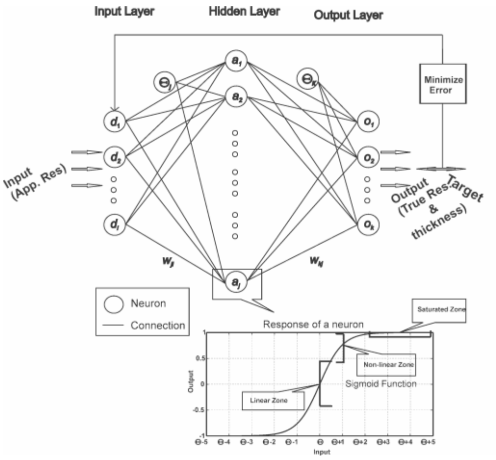

Figure 3: Network topology of artificial neural network

In the Bayesian framework, a suitable prior distribution, say $P(w)$ of weights is considered before observing the data instead of single set of weights. Using Bayes rule, *a posteriori* probability distribution for the weights, say $P(w|s)$ can be given as (Bishop, 1995),

$$P(w\,|\,s) = \frac{P(s\,|\,w)P(w)}{P(s)} \quad \text{....................(6)}$$

Where, $P(s\,|\,w)$ is the data set likelihood function, and the denominator, $P(s)$ is a normalization factor. The denominator $P(s)$ is intractable so direct estimation of posterior $P(w\,|\,s)$ is impossible. Using the rules of conditional probability, the distribution of outputs for a given input vector, x can be written in the form (Bishop, 1995),

$$P(x\,|\,d,s) = \int P(x\,|\,d,w)P(w\,|\,s)dw \quad \text{........(7)}$$

The chief problem in Bayesian computation is evaluating integrals for posteriori pdf of weights (equation 6) and network output (equation 7). Fortunately numerical methods Monte Carlo combine Markov Chain (MCMC) sampling based method plays an important for evaluating posterior integrals. The equation (7) can be approximated as

$$P(x\,|\,d,s) = \frac{1}{N}\sum_{n=1}^{N} P(x\,|\,d,w_m) \quad \text{........... (8).}$$

Where $\{w_m\}$ represents a MCMC sample of weight vectors obtained from the distribution $P(w\,|\,s)$ and N is the number of points w sampled from $P(w\,|\,s)$.

Hybrid Monte Carlo (HMC)

The HMC based algorithm is a sampling algorithm that takes into consideration certain gradient information. The algorithm follows the following sequence of steps once a step size θ and the number of iterations L have been decided upon: First randomly choose a direction τ : τ can be either -1 or +1 with the probability 0.5 simulate the dynamics forward or backward in time. The transition probability matrix satisfies microscopic reversibility which means that probability of these two transitions from q_j to q_i or from q_i to q_j be same at all times and each pairs of points maintains a mutual equilibrium. Then carry out the iterations: starting with the current state $[q,\ p]$ = $[(q(0),\, p(0)]$ of energy H , where p is a momentum term which is randomly evaluated at each step, let algorithm perform L steps with a step size of θ resulting in the candidate state, $[w^*,\ p^*]$ with energy H * .The candidate state is accepted with usual metropolis probability of acceptance, $\min\{1, \exp[-(H^* - H)]\}$, where $H(.)$ is the Hamiltonian energy. If the candidate state is rejected then the new state will be the old state. These three steps, in essence, describe how the sampling is done from posterior distribution of network parameter so that the summation of equation (7) can be accomplished and the posterior distribution can be found and thus allowing the optimization of the network. The momentum term p can be randomly generated or it can be changed dynamically at each step and there are different ways of doing this (Bishop 1995).

Examples

We demonstrate the applicability of the proposed inversion method on the actual DC resistivity data. We inverted 30 number of DC resistivity sounding data using the newly developed Hybrid Monte Carlo-based Bayesian Neural

Network (HMC-BNN) approach (following Maiti et al., 2011). One dimensional inversion results of some selected sounding station are shown (Figure 4). The figure shows the comparison of HMC-BNN based inversion results with The Singular Value Decomposition (SVD)-based conventional DC resistivity inversion results.

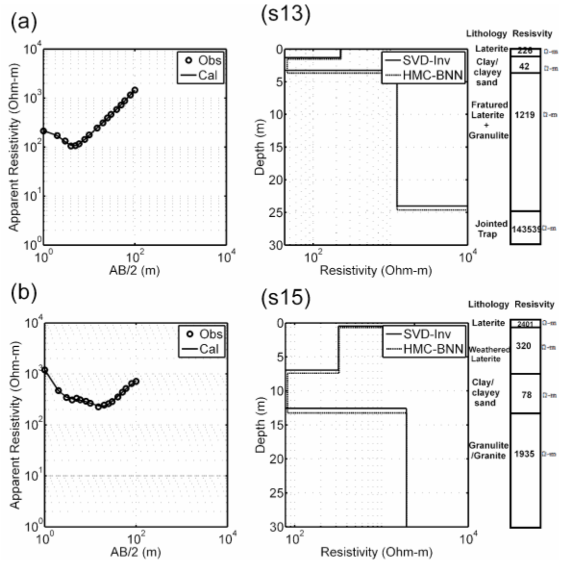

Figure 4: VES data inversion and lithology correlation for sounding station 13 and 15.

In figure 4, ‘Obs’ denotes observed, and ‘Cal’ denotes the calculated resistivity values, ‘SVD-Inv’ denotes SVD based conventional resistivity inversion results, ‘HMC-BNN’ denotes Hybrid Monte Carlo-based Bayesian Neural Network results. The new HMC-BNN inversion solution is well correlated with SVD-based conventional inversion solution. Additionally the HMC-BNN approach provides uncertainty estimation, which is essential for geophysical interpretation. In the present analysis we found an average uncertainty estimate of about ~0.31. We also note that the BNN based approach avoid “oscillation” problem around the many local minima while network training/optimizing. The final solution presented here is mean of thousands equally probable model simulation (Maiti et al., 2011). The HMC-BNN results lies within the 90% confidence interval (CI) attesting the physical significance of results. We note that majority of the Schlumberger sounding curves in the area are H-type (86%), Q-type (7%) and rest are A-type (7%). The apparent resistivity (ρ_a) vs half of the current electrode separation (AB/2) on log-log graph suggests that three to four layered structured in this area (Figure 4).

Two dimensional modeling

Two-dimensional inversion results suggest that the surface layer is seldom thick, being only 1-3 meters. The surface layer is thinner over the elevated land/hill and in some places it is absent too. The second layer has a low resistivity, which might be indicative of weathered rock (mostly, laterite to fractured basalt) possibly saturated with water. Its thickness varies from 10-20m. Third layer is comparatively less resistive (clays, clayey sand) than the crystalline basement rock of granite/granulites. It is also observed that generally the rocks in the lower ground and hill slope are more weathered than those in the high ground, although there are some exceptions too. In our present study 2-D model based results appears to more appropriate to infer lateral resistivity variation caused by multiple episodes of lave flows and genesis than the one dimensional results.

It is noteworthy that the present BNN based resistivity model is considerably stable even in the presence of correlated red noise (Figure 5), and hence provide some basis to apply the algorithm for modeling the resistivity data collected in noisy geological environment. One can see from Figure 5 that there is a prominent resistivity low (~2-15 Ohm-m) below the station s30, Nivti. This conductive feature indicate the possible presence of salt water intrusions into the fresh water zone along the profile AA’. In between S28 and S25 along the profile CC’, there is a prominent groundwater reservoir (Figure 6). The borehole lithology is supplemented by well number 6 (w6). The fractured/weathered granitic rock saturated with water is extended in the NE-SW direction along Zaraph-Vengurla up to the depth of 8-13m which might serve as conduits. It may be noted that in the fractured rocks chemical weathering is almost absent. The semi weathered and weathered rocks are partly jointed or fractured in this area (Figure 6).

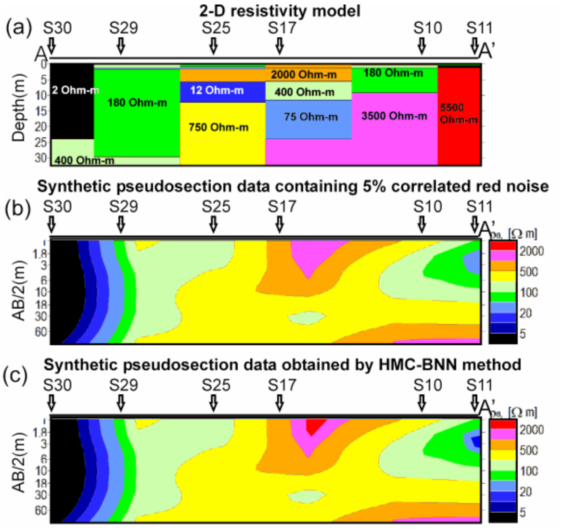

Figure 5 :(a)Synthetic training model(b) pseudosection prediction by HMC-BNN when data is noisy(c) HMC- BNN model for noise free data.

In weathered rocks both inter-granular and fracture porosities exist. However, in fractured rock (not weathered) only fracture porosity occur. The presence of feldspar and clay in the fractured rock reduces the permeability of the rock to some extent (Verma et al., 1980).

The aquifer system is partly composed of clayey sand layer and highly fractured weathered granites/granulites. Among the geological formations overlying the aquifer system, the clay layer displays the lowest resistivity range (Figure 4) which is very common in the area near to the west coast of Arabian Sea. Clay minerals have a high cation exchange capacity and because of their proportion in weathered crystalline rock terrains, yield a low electrical resistivity. We note that interpretation of resistivity data is ambiguous resulting in different combinations of resistivity and thickness. It is also important to note that that VES sees difficulty in recognizing very thin layers, which can sometimes contains water should be treated with caution (Zohdy, 1978).

In addition to the finer mapping and demarcating the zones of weathered rocks, lineaments and fractures etc in the crystalline basement, the present analysis is also able to define the line of saltwater intrusions along the Nivti-Vengurla coast of the area, which is of considerable significance for groundwater exploration. Further this result is useful to gain the better insights of the complex geology of different intrusions in the hydro-geological system in the area.

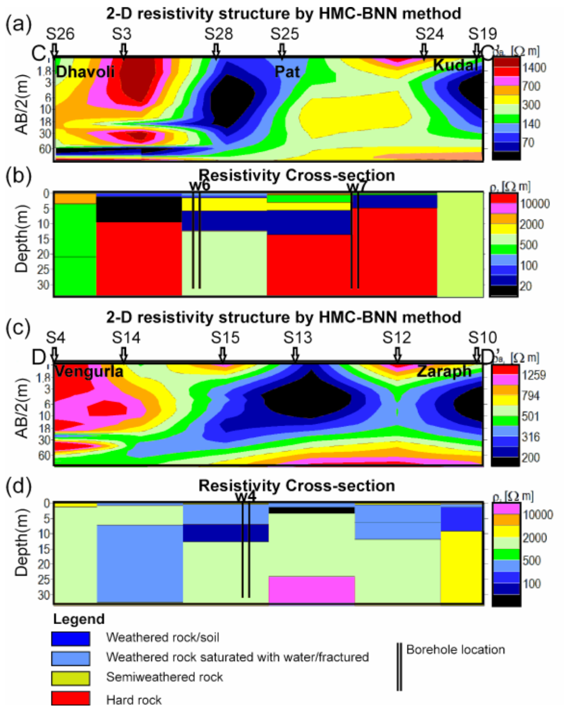

Figure 6: Electrical structure along profile (a-b) (CC') and profile (DD')

Conclusions

A novel Hybrid Monte Carlo-based Bayesian neural network inversion scheme is developed for the interpretation of DC resistivity data. This method, on contrary to the other inversion scheme, does not require a priori information on initial model for optimization. Additionally, the new approach also provides an opportunity to estimate uncertainty measures, which is essential for solving geophysical inverse problem. The present method is a sampling-based global inversion scheme, which takes care of non-linearity and non-uniqueness in a natural way. Besides providing some new and detail information, the present results also corroborate well with existing borehole litho logical results. The new Hybrid Monte Carlo-based Bayesian Neural Network approach offers inversion results instantaneously. In many geophysical application (e.g., seismic, electrical tomography), while repeated inversion is required from same prior information, the new approach could be more feasible and prospective in this regard.

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