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Singular Value Analysis (SVA) of electrical response of a General Aquifer Model (GAM) for direct and inverse relation between electrical resistivity and hydraulic conductivity

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Summary

Using three layer general aquifer model (GAM) it has been established that the most dominant parameter for guiding the direction of the current at basement aquifer interface and why direct and inverse relations exist between hydraulic conductivity and electrical resistivity. Software package Matlab is used for SVA that has fixed the most dominant parameter, transverse electrical unit resistance, in case of conducting third layer and longitudinal electrical unit conductance, in case of resistive third layer of GAM. Two examples of hydraulic parameter estimation from electrical parameter are included as examples.

Introduction

For groundwater management simulation and modelling are required to understand the hydrodynamics. In this connection it is interesting to recall the statement of John H. Cushman "that over the last two decades, researchers who have been trying to understand the transport processes in porous media have begun to recognize that we do not know nearly as much we once thought we did. Natural porous media are generally not homogenous, nor are they uniformly random…... Transport in porous media is truly multi-disciplinary science that requires tools and concepts from many fields (Schon 1996)."

The hydrodynamic and electrodynamics of a homogeneous continuum are well understood, however, the same can't be said about porous media. The modern day forecast theory for hydrodynamics are cast in stochastic framework and demand knowledge of the stochastic properties of hydraulic conductivity of the field which is a high cast affair in conventional sense, if number of locations are sufficiently large. However, it can be circumvented with estimation of hydraulic conductivity from surface geoelectrical measurements. Physical condition (tortuosity, porosity and specific surface) controlling the electrodynamics likewise controls the hydrodynamics. The fact that the surface geoelectrical measurements can be made accurately and inverted reasonably, the electrical resistivity of the aquifer can be converted to hydraulic conductivity provided that physically tenable relationships between electrical and hydraulic transport properties of the aquifer are available.

It has been practice in well logging to estimate permeability from electrical log data through Archie (1942) and Kozeny-Carman equations, however for surface measurements the existing relationship between inverted electrical properties from surface geoelectrial measurement and hydraulic transport property determined through pump test data are mostly empirical in nature. Unfortunately, these empirically derived relations shows both direct as well as inverse relationship between electrical resistivity and hydraulic conductivity presenting a paradoxical situation and hamper the confidence level in estimated values of hydraulic parameter from surface geoelectrical measurements. As examples we have selected seven such data sets available in literature (Kelly, 1977; Kosinky and Kelly, 1981; Yadav, 1995; Ranjan, 2006; Frohlich et al, 1996; Kumar et al, 2001; Chandra et al, 2008). We are not distracted by the empirical relationship determined in these publications and have taken only the interpreted electrical and hydraulic parameters and presented them graphically in Figure 1 and Figure 2. Figure 1 contains data from Frohlich et al (1996), Kumar et al (2001), and Chandra et al (2008) and all these data show a common inverse trend between electrical resistivity and hydraulic conductivity although the areas

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from where these data belongs are geographically far apart. Similarly Figure 2 includes data of Kelly (1997), Kosinky and Kelly (1981), Yadav (1995), and Ranjan (2006) shows a common direct trend between electrical resistivity and hydraulic conductivity. These presentations confirm that the physical conditions under which electrical and hydraulic transport are taking place is more important than the geographical contiguity of the area. Interestingly the data shown in Figure 1, on the basis of the discussions provided by respective authors indicate a geological terrain where the layer below the aquifer is comparatively electrically resistive where as those shown in Figure 2 indicate that the layer below aquifer is relatively conducting.

Sri Niwas and Singhal (1981, 1985) developed two theoretical equations combining Darcy,s law of fluid flow and Ohm's law of current flow in a unit prism of aquifer material given by

$$\boldsymbol{T_h} = \alpha \mathbf{R};\ \boldsymbol{K_h} = \alpha\rho \qquad (1)$$

$$\boldsymbol{T_h} = \beta \mathbf{R};\ \boldsymbol{K_h} = \beta/\rho \qquad (2)$$

Where T_h is the hydraulic transmissivity, is hydraulic conductivity, ρ is the electrical resistivity, R is transverse electrical unite resistance and C is the longitudinal electrical unite conductance of the aquifer. α and β are the constant of proportionality. Equation (1) is derived by considering a vertical flow of the electric current resulting a direct relation between hydraulic conductivity and electrical resistivity whereas equation (2) is derived by considering a horizontal flow of the current showing an inverse relation. The hydraulic flow is assumed to be predominantly in horizontal direction. In a subsequent publication Sri Niwas and Lima (2003) and Sri Niwas et al. (2011) explained the physical basis for the existence of the two equations through rigorous deduction considering a three layer GAM (Sri Niwas and Lima, 2006). The objective of the present paper is to provide more physical insight to the dominant nature of R and C of the aquifer through singular value analysis of the inverse problem for GAM for firming up the physical reason for direct and inverse relationship between electrical resistivity and hydraulic conductivity of aquifers. Also the paper includes the applicability of equations (1) and (2) for estimation of hydraulic parameter from electrical parameter for one set of data from each group.

Geoelectrical Modelling

The mathematical relation for calculating apparent resistivity over an N-layered earth in Schlumberger configuration is given by (Koefoed 1979)

$$\boldsymbol{\rho_{as}} = \boldsymbol{s^2} \int_0^\infty [\boldsymbol{T}(\boldsymbol{\rho_i}, \mathbf{d}_i; \boldsymbol{\lambda})]\, \mathbf{J_1}(\boldsymbol{\lambda s})\boldsymbol{\lambda d\lambda}\ ;\ \mathbf{i=1,2,\ldots N} \qquad (3)$$

Where ρ_i and d_i are resistivity and thickness of ith layer, $J_1(\lambda s)$ is the Bessel function of first order and first kind, λ is the integration variable and s is the half current electrode separation. The function within the large bracket is the resistivity transform function, T(λ) .In the case of multi-aquifer sequence, the N-layered earth can be represented as a 3-layer GAM by combining the intermediate aquifer layers (from i=2 to N–1) as a single pseudo-isotropic layer of resistivity ρ and thickness *d*. For this 3-layer GAM (having resistivity ρ_1, ρ, ρ_3 and thickness d_1, d), equation (3) is reduced, for significantly large half current electrode separations relevant to the aquifer layer (Sri Niwas and Lima 2006), as

$$\boldsymbol{\rho_{as}} = \boldsymbol{s^2} \int_0^\infty \frac{\rho_3 + \lambda(R_1 + R)}{1 + \lambda\rho_3(C_1 + C)}\, \mathbf{J_1}(\boldsymbol{\lambda s})\boldsymbol{\lambda d\lambda} \qquad (4)$$

where R_1 ($\rho_1 d_1$) and $\boldsymbol{R}$ (=ρ d) are the transverse unit resistances of the top layer and middle aquifer layer respectively, C_1(=d_1/ρ_1) and $\boldsymbol{C}$ (=d/ρ) are the longitudinal unit conductance of top and aquifer layers respectively, and ρ_3 is the resistivity of the half space below aquifer layer. Equation can be posed as a quasi- linear inverse problem as

$$\Delta \mathbf{y} = \mathbf{G}\Delta \mathbf{x} \qquad (5)$$

where Δy = n x 1 column vector composed of difference between observed resistivity ($\rho_{as\,i}$) and computer developed resistivity ($\rho^0_{as\,i}$) using a starting model, Δx is the m x 1 column vector composed of difference between layer parameter of starting model and sought for model and matrix *G* of order n x m is referred as Jacobean defined as $G_{ij} = \frac{\partial \rho_{as\,i}}{\partial x_j}$. Using Singular Value Decomposition (SVD) (Lanczos, 1961) matrix G can be decomposed as product of three matrices

$$G = U\Lambda V^T$$

The vectors contained in the matrix U are data eigenvector and vectors contained in the matrix V are parameter eigenvectors and diagonal matrix Λ contains eigenvalues of matrix G in decreasing order of magnitude.

According to Inman et al. (1973) parameter eigenvectors represent the independent linear combinations of the parameter differences that can be determined from the independent linear combinations of the data points as given by the data eigenvectors. Parameters associated with the largest eigenvalue are found most accurately determined, easily resolvable and most dominant whereas those parameters associated with the smallest eigenvalue are the least accurately determined, least resolvable and weakly present in the linear system.

According to Koefoed (1979), the effect of change of on a resistivity transform function is similar to that of change of half electrode spacing (*s*) on an apparent resistivity function. So in place of ρ_{as} and *s* we have used T (λ) and 1/ λ for discussion.

Parameter eigenvector (V) of Jacobean G will be same as eigenvector of square matrix (G^TG). Parameter eigenvector associated with each eigenvalue for highly resistive and conductive half space are plotted at arbitrary constant spacing (figure 3-4, Appendix). Further parameter eigenvector associated with largest eigenvalue will be most important for deciding the dominant parameter responsible for guiding the direction of current at the aquifer-basement interface.

Numerical Experiment

In case of highly resistive half space (Figure 3, Appendix) parameter eigenvector associated with largest eigenvalue indicates longitudinal conductance *C* is most resolvable and dominant parameter for determining the value of apparent resistivity. Parameter eigenvector associated with smallest eigenvalue indicates that transverse resistance *R* of second layer is least resolvable and weakest parameter. Whereas if half space is highly conducting (Figure 4, Appendix) parameter eigenvector associated with second highest eigenvalue suggest that transverse resistance *R* is most resolvable and dominant parameter for determining the value of apparent resistivity. Parameter eigenvector associated with lowest eigenvalue indicates that longitudinal conductance *C* is least resolvable and weakest parameter in case of conducting half space. It is interesting to recall the mathematical results of Sri Niwas et al (2011) where it is shown that in case of insulating half space the apparent resistivity is solely determined by *C* and in case of perfectly conducting half space it is exclusively determined by R. According to Maillet (1947) if one considers a geological column built on a square unit, *R* measures the resistance to the lines of current perpendicular to the strata, and *C* is the conductance offered to the lines of current parallel to strata. In the light of this statement, mathematical modelling results of Sri Niwas et al (2011) and physical insight provided by SVD results it is established that the applicability of equations (1) and (2) is constrained by physical situation whether the layer below the aquifer is conducting or resistive. On this basis the data of Figure 1 can be evaluated through equation (2) whereas that of Figure 2 can be analysed by equation (1).

For SVA we have selected a robust 3-layer GAM with layer parameter as: ρ_1= 150 *Ohm m,* ρ = 80 *Ohm m* and ρ_3 = 10,000.0 *Ohm m* (model 1) and 0.001 *Ohm m* (model 2) so that the resistive as well as conductive half-space is simulated for the same upper two layers of thickness d_1 = 5 *m*, d = 6 *m*. Figure 3 presents SVA for model 1 and that Figure 4 for model 2. It is apparent that out of the two Dar-Zarrouk parameters (Maillet, 1947) *C* and *R* of aquifer, in model 1 the dominant parameter is *C* and that in model 2 it is *R*. This clearly establishes that equation (1) should be for model like 2 whereas equation (1) is suitable for model like 1. In this background we have reanalysed the data of Frohlich et al (1995) and Ranjan (2006) through equation (2) and (1) respectively with β = 42.5 (Figure 5) and α = 0.33 (Figure 7). Theoretically computed hydraulic parameters are cross-plotted with observed values in Figures (6) and (8) respectively. It is significant that RMS error between computed and observed values in case of Figure 6 is 0.06 *m/day* whereas in case of Figure 8 the RMS error is 121 m^2/day.

Conclusion

On the basis of results developed in this paper and in the light of results obtained earlier for direct and inverse relation between electrical resistivity and hydraulic conductivity we can conclude that in cases of *A-* ($\rho 1 < \rho < \rho_3$) and *H- type* ($\rho 1 > \rho < \rho_3$) sounding curves for 3-layer GAM equation (2) is appropriate and in cases of *K-* ($\rho 1 < \rho > \rho_3$) and Q-type ($\rho 1 > \rho > \rho_3$) curves equation (1) is recommended

Appendix

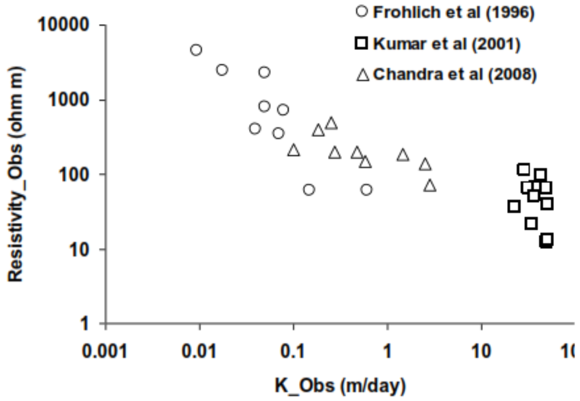

Figure 1

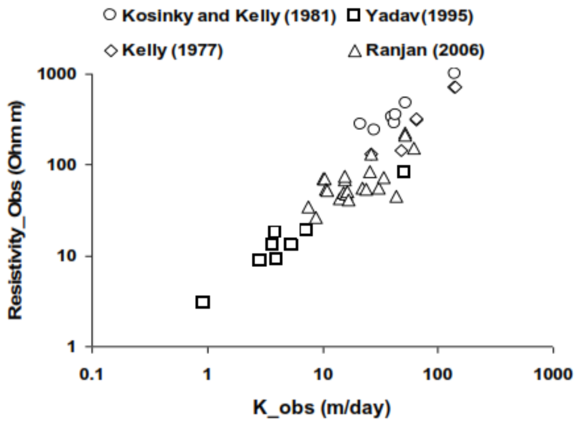

Figure 2

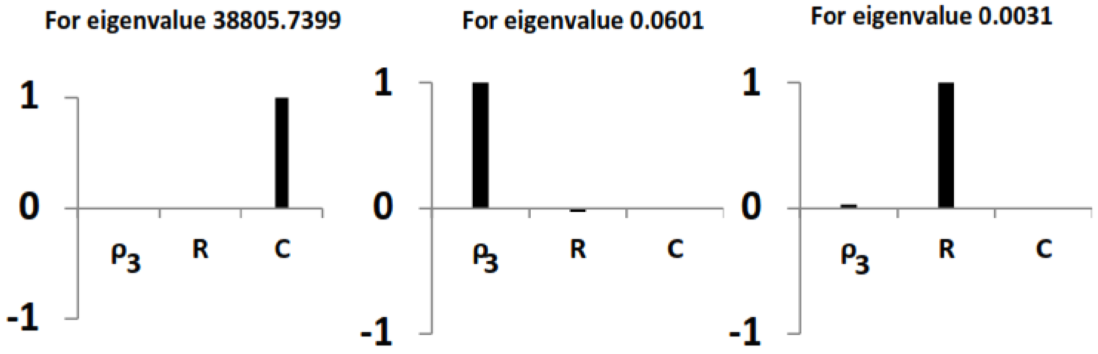

Figure 3

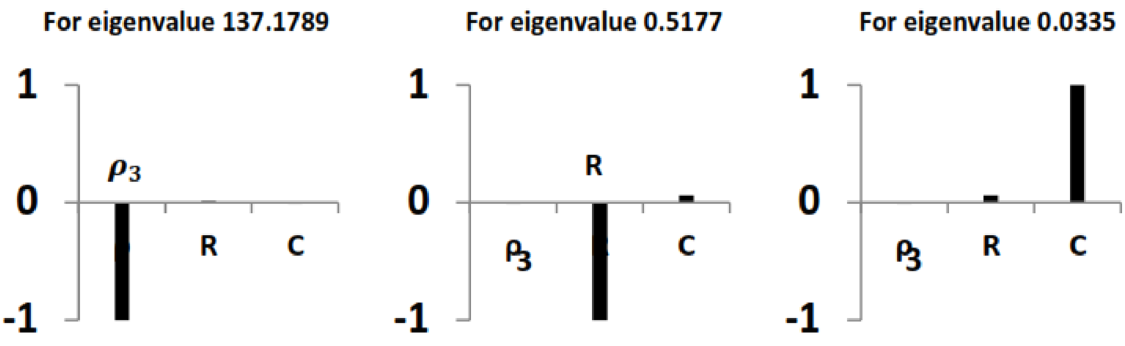

Figure 4

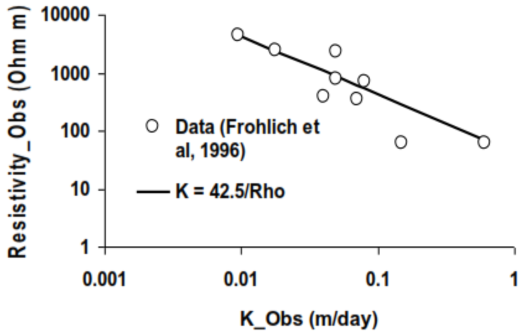

Figure 5

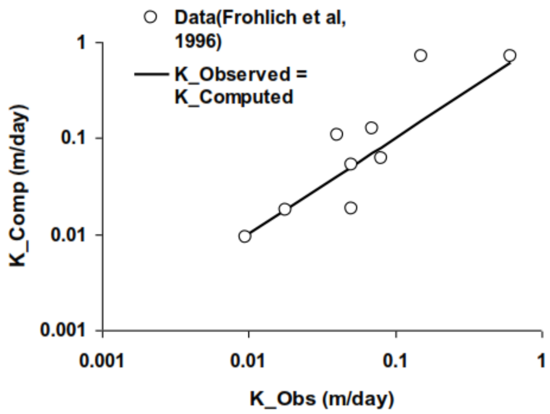

Figure 6

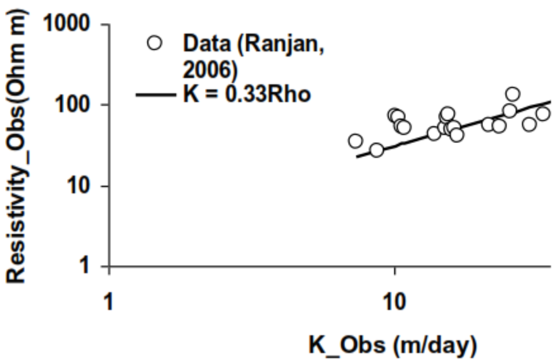

Figure 7

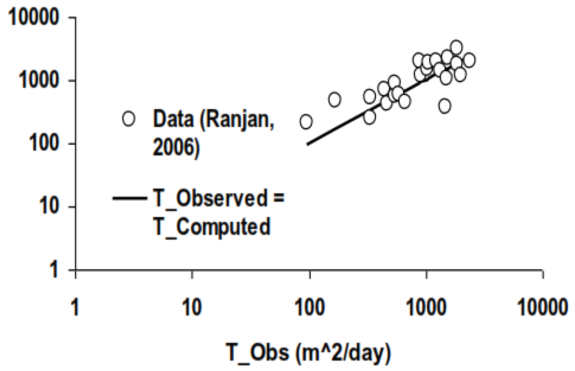

Figure 8

Caption of Figures

Figure 1: Showing direct relationship between Electrical resistivity and Hydraulic Conductivity

Figure 2. Showing inverse relationship between Electrical resistivity and Hydraulic conductivity

Figure 3. Singular value analysis of model 1 with layer parameters: $\boldsymbol{\rho_1}$ = 150 ohm.m, $\boldsymbol{d_1}$ = 5 m, $\boldsymbol{\rho_2}$ = 80 ohm.m, $\boldsymbol{d_2}$ = 6 m, $\boldsymbol{\rho_3}$ = 10000 ohm.m

Figure 4. Singular value analysis of model 1 with layer parameters: $\boldsymbol{\rho_1}$ = 150 ohm.m, $\boldsymbol{d_1}$ = 5 m, $\boldsymbol{\rho_2}$ = 80 ohm.m, , $\boldsymbol{d_2}$ = 6 m, $\boldsymbol{\rho_3}$ = 0.001 ohm.m

Figure 5. Analysis of Frohlich et al (1996) data using equation (2).

Figure 6. Cross-plot between computed K and observed K values.

Figure 7. Analysis of Ranjan (2006) data using equation (1).

Figure 8. Cross plot between computed T and observed T values.

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