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Nonlinear one-dimensional seismic waveform inversion using Harmony search

Vivek Kumar*, Prof. Pravin Kumar Gupta, *IIT Roorkee*

Summary

Seismic inverse problems include minimization of error between the observed and synthetic seismograms to obtain the model parameters that will provide information about the subsurface. Generally we face two problems: 1) the model space is very large and 2) the error function is multimodal. Existing calculus based methods are local in scope and easily get trapped in local minima of error function. Other methods such as simulated annealing, genetic algorithm and harmony search can be applied to solve such global optimization problems and these don't depend on starting model and do not require any gradient information. All these methods bear analogy to natural system and are robust in nature. e.g. - the recently developed harmony search algorithm is based on musical process of searching for a perfect state of harmony, where a group of musicians together play a pitch with their instrument to obtain the best harmony. Musicians follow three rules in playing a pitch: 1) Play any pitch from his memory. 2) Play any of adjacent pitch from his memory. 3) Play totally random pitch within range. This process is repeated until they together obtain the best harmony.

To use Harmony Search (HS) efficiently for 1D seismic waveform inversion, we require a modeling method that rapidly performs forward modeling calculations and two parameters - Pitch Adjusting Rate (PAR) and Band Width (BW), which will enable us to find the global minimum of error function rapidly. With the help of computers, reflectivity method proved to be fastest if only plane wave seismogram is required. Hence, the principal problem with harmony search is that it requires a proper band width and pitch adjusting rate. By initiating harmony search with fixed parameter values, such as number of variable, Harmony Memory Size (HMS), Harmony Memory Considering Rate (HMCR), PAR and BW. We noticed that with small BW and high PAR, we reach very close to global minimum error rapidly. We have applied this technique successfully to band limited synthetic data in presence of random noise. In most cases we find that we are able to obtain very good solution using the plane wave seismogram.

Keywords*: Global optimization, Meta-heuristics, Harmony Search, Reflectivity Method, Weighted Least Square*

Introduction

There exist several methods for inversion to determine subsurface properties using recorded seismic data. For prestack data that exploit offset dependencies in seismic amplitudes and travel times, the waveform inversion methods can be broadly classified in two categories: (1) the direct inversion methods based on Schur algorithms/layer-stripping principle (e.g., Clarke, 1984 [1]; Yagle and Levy, 1985 [2]) and (2) iterative inversion schemes or nonlinear least-square inversion methods (e.g., Tarantola , 1987 [3]; Pan et al., 1988 [4]). Direct problem generally suffers from many fundamental limitations, for example, the method becomes unstable with noisy data; the error is additive and accumulates with depth. Nonlinear least squares inversion methods have been discussed in great detail (see, e.g., Tarantola and Vallette, 1982 [5]). Pan et al. (1988) [4] and Pan and Phinney (1989) [6] used the nonlinear least-squares inversion method based on Tarantola's formulation for full waveform inversion of plane-wave seismograms in stratified acoustic media. The principal problem with this method is that it requires a good starting model since it looks for a solution in neighborhood of starting model. Since the error energy function that we minimize has many extrema, if the trial solution is too far from the global

F-15, Cautley Bhawan, IIT Roorkee, Roorkee-247667
Email-vivekke970@gmail.com

minimum, the method may converge to a local rather than the global minimum. In contrast to the direct inversion methods, these methods are multiparameter optimization procedures and any physical phenomena observed in the data that are not modeled serve only to increase the residual error.

The alternative to these methods is enumerative scheme in which each point in model space is searched sequentially. This, however, is an impossible task for our model space is extremely large. Methods that are a compromise between nonlinear least square methods and the enumerative scheme can be defined which use a random search and a rule to help guide their form of search. These methods don't depend on starting model and don't require any derivative information. Simulated annealing, Genetic algorithm and Harmony search generally belongs to this category. In this paper we apply Harmony search algorithm for inversion of plane wave seismogram and compare it with simulated annealing and genetic algorithm. For this purpose a prototype program was designed in C sharp.

Harmony Search Algorithm

Recently, Geem et al.(2001) [7] developed a new Harmony Search (HS) meta-heuristic algorithm that was conceptualized using natural musical performance process of searching for perfect state of harmony, such as during jazz improvisation.. The harmony in music is analogous to optimization solution vector and the musician improvisation is analogous to local and global search scheme in optimization techniques. More detailed description of this algorithm can be found in Geem et al. 2005[8], Mahdavi et al. 2007[9], Geem 2005[10] and therefore only a brief summary is given here.

Figure-1 shows analogy between music improvisation and optimization[7]. In music improvisation each player sounds any pitch within the possible range, together making one harmony vector. If all the pitches make a good harmony, that experience is stored in each player's memory and the possibility to make a good harmony is increased next time. Similarly in optimization, each decision variable initially chooses any value within the possible range, together making one solution vector. If all the values of decision variables make a good solution, that experience is stored in each variable's memory, and the possibility to make a good solution increases next time. Step-2- shows the structure of harmony memory (HM) that is the core part of HS.

In real optimization, each musician can be replaced with each decision variable (like density, velocity, thickness) and its preferred sound pitches are replaced with each decision variable's preferred values (like velocity {1 to 4km/s}, density {1.2 to 3gm/cc}, thickness {0 to 0.5km}). And if first variable is choses {2km/s} out of {1 to 4 km/s}, second {2gm/cc} out of {1.2 to 3 gm./cc} and third {0.3km} out of {0 to 0.5km}, those values {2km/s,2gm/cc,0.3km} make another solution vector. And if first variable is choses {2km/s} out of {1 to 4 km/s}, second {2gm/cc} out of {1.2 to 3 gm./cc} and third {0.3km} out of {0 to 0.5km}, those values {2km/s,2gm/cc,0.3km} make another solution vector. And if this new vector is better than existing worst vector in the HM, the new vector is included in the HM and worst vector is excluded from the HM. This procedure is repeated until certain termination criterion is satisfied.

When a musician improvises one pitch, usually he follows any one of the following three rules:

1. Playing any one pitch from his memory
2. Playing an adjacent pitch of one pitch from his memory
3. Playing totally random pitch from possible sound range.

Similarly, when each decision variable chooses one value in HS algorithm it follows any one of following three rules:

1. Choosing any one value from HM (defined as HMCR)
2. Choosing adjacent value of one value from HS memory (defined as PAR)
3. Choosing totally random value from possible value range (defined as randomization)

These three rules in HS are effectively directed using two parameter HMCR and PAR (like crossover and mutation in genetic algorithm). Below we represent steps for Harmonic search algorithm

Figure-1: Analogy between music improvisation and engineering optimization

The steps in the procedure of harmony search are shown in Figure-2. These are as follows:

Step 1 Initialize the problem and algorithm parameters.
Step 2 Initialize the harmony memory.
Step 3 Improvise a new harmony.
Step 4 Update the harmony memory.
Step 5 Check the stopping criterion.

Step-1- Includes defining our error function f(x) where x is solution vector $x^0 = (v_1, t_1, v_2, t_2 \ldots\ldots\ldots v_n, t_n)$
Where, v_i is velocity $1<i<n$; t_i is thickness $1<i<n$; n is number of layers.

Step-2- Includes filling of HM matrix with random numbers generated between the range given for velocity (0-5km/s) and thickness (0-0.5 km). This HM represents model parameter in our 1D inversion problem.

$$HM = \begin{bmatrix} v_1^1 & t_1^1 & v_2^1 & t_2^1 & & v_n^1 & t_n^1 \\ v_1^2 & t_1^2 & v_2^2 & t_2^1 & & v_n^2 & t_n^2 \\ v_1^3 & t_1^3 & v_2^3 & t_2^1 & \ldots\ldots\ldots & v_n^3 & t_n^3 \\ v_1^{HMS} & t_1^{HMS} & v_2^{HMS} & t_2^{HMS} & & v_n^{HMS} & t_n^{HMS} \end{bmatrix}$$

Where, v_i is velocity $1<i<n$; t_i is thickness $1<i<n$; n is number of layers and HMS represents the number of solution stored in HM.

Figure-2: Flow chart of classical Harmony Search Algorithm

Step-3- A new harmony vector, $x^1 = (v_1, t_1, v_2, t_2 \ldots\ldots\ldots v_n, t_n)$ is generated based on above three rules. In the memory consideration, the value of the first decision variable for the new vector is chosen from any of the values in the specified HM range v^1, $v^2 \ldots v^{HMS}$, values of the other decision variables are chosen in the same manner. The HMCR, which varies between 0 and 1, is the rate of choosing one value from the historical values stored in the HM, while (1-HMCR) is the rate of randomly selecting one value from the possible range of values. Every component obtained by the memory consideration is examined to determine whether it should be pitch-adjusted. This operation uses the PAR parameter. If the pitch adjustment decision for x_i is yes, x_i is replaced as follow:

$$x_i' = x_i' \pm rand() * bw$$

Where,
bw is an arbitrary distance bandwidth
rand( ) is a random number between 0 and 1

Step-4- If the new harmony vector, is better than the existing worst harmony vector in the HM, judged in terms of the objective function value, the new harmony is included in the HM and the existing worst harmony is excluded from the HM. Hence, each iteration improves our model parameter in HM by removing the worst row with new best row.

Step-5- If the stopping criterion (maximum number of improvisations) is satisfied, computation is terminated. Otherwise, Steps 3 and 4 are repeated.

Forward Modeling

The Harmony search method requires the evaluation of the forward problem a large number of times. Thus a modeling method that is accurate and that can be computed rapidly is needed. Although the finite difference of a 3-D wave equation (Reshef et al., 1988[11]) may be used, the computation is too intensive to be practical for seismic inversion in which many thousands of iterations are needed. The reflectivity method (Kennett, 1983[12]) requires seismic data to be converted from the timedistance domain into the frequency wavenumber domain. When the number of recording channels is limited, aliasing may affect the quality of inversion. In this paper, we use a convolutional model to generate synthetic data. We note that a similar forward modeling approach was also used by Sen and Stoffa (1991) [13] in their nonlinear least-squares inversion. Since we will model only the plane-wave seismograms, we must first do a plane-wave decomposition of the observed seismic data as outlined, for example, by Brysk and McCowan (1986)[14]. It is assumed that in laterally homogeneous acoustic media, prestack seismic data can be approximated by convolution of offsetdependent reflection coefficients with a known wavelet. The convolutional model assumes plane-wave propagation across the boundaries of horizontally homogeneous layers, and takes no account of the effects of geometrical divergence, anelastic absorption, wavelet dispersion, transmission losses, mode conversions, and multiple reflections. For a convolution model to be valid, the seismic data must be processed to eliminate those effects and to restore plane-wave amplitudes of primary P-wave reflections. We only use a few well-distributed plane-wave seismograms, e.g., 4 to 8 over the range 0 to 0.5s/km. (Sen and Stoffa (1991) [13])

Objective function

In this paper, we use an L2-norm error function as our objective function. A similar type of objective function is also used in Xin-Quan Ma (2002) [15]. This represents the least square deviation between the observed and modeled offset seismic gather. In order to constrain the impedance models, we add a priori impedance and a priori misfits. The objective function is expressed as

$$\Delta f = W1 \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\left\| S_{obs}^{ij} - S_{mod}^{ij} \right\|}{\sum_{i=1}^{m}\sum_{j=1}^{n}\left\| S_{obs}^{ij} \right\|} + W2 \frac{\sum_{i=1}^{n}\left\| I_{obs}^{i} - I_{mod}^{i} \right\|}{\sum_{i=1}^{n}\left\| I_{obs}^{i} \right\|} + w3 \frac{\sum_{i=1}^{n}\left\| V_{obs}^{i} - V_{mod}^{i} \right\|}{\sum_{i=1}^{n}\left\| V_{obs}^{i} \right\|} + w4 \frac{\sum_{i=1}^{n}\left\| T_{obs}^{i} - T_{mod}^{i} \right\|}{\sum_{i=1}^{n}\left\| T_{obs}^{i} \right\|}$$

Where, S_{abs}^{ij} is the observed seismic amplitude at time index i and channel index j: S_{mod}^{ij} is the synthetic seismic amplitude at time index i and channel index j; I_{abs}^{i} is an a priori P-impedance trend at time index i; I_{mod}^{i} is the modeled P-impedance at time index i; V_{abs}^{i} is an a priori P-velocity at time index i; V_{mod}^{i} is the modeled P-velocity at time index i; T_{abs}^{i} is an a priori Thickness; T_{mod}^{i} is the modeled thickness at time index i; n is the number o samples in a seismic trace, m is the number of channels in aseismic gather, and W1, W2, W3 and W4 are weights applied to the three terms, respectively. Since the three terms in the objective function have been normalized, the weighting factors can be chosen as W1 =W2=W3=W4=1 inmost cases.

Numerical Examples

The aim of our inversion scheme is to estimate the velocity, density and thickness of different layers comprising the 1D earth model. To illustrate the harmony search process we show the inversion of noise-free band-limited synthetic data for eight-layer acoustic earth model (True values are represented in Table-1). Figure-3a shows noise free (Γ-p) seismogram generated for frequency band of 10-80 Hz at five equispaced ray parameter values in the range 0-0.5 sec/km for an 8 layer earth model. We search for velocity, density and thickness for each layer within a priori information range for each layer. Figure-3b represents corresponding decrease in error function. Figure-4a and 4b is case for 30% noisy data. We note here that harmony search does not require any trend information and does not depend on starting model. The search intervals for this example were chosen so as not to make the computation expensive. We start from random location in model space and initiate the inversion following the algorithm.

One benefit of HS is that it can estimate the low velocity zones at great depth, since user has privilege to adjust the range of each parameter value for each layer. However, we are faced with problem of choosing a proper Pitch Adjusting Rate (PAR) and Band width (BW). Originally fixed parameter values were used. However, some researchers proposed changeable parameter values. Mahdavi et al (2007) [9] suggested that PAR increases linearly and BW decreases exponentially with iteration. Geem [16] tabulated fixed parameter values. In our case for synthetic seismic data, we obtain a relationship between PAR, BW and iteration and found the result was following the case defined by Mahdavi for best fitness value. After large number of experimentation done by varying PAR from 0.25 to 0.65 with a fixed iteration, it is found that the converged iteration increases linearly as we increase the PAR. Similarily for varying BW from 0.2 to 0.02 with a fixed iteration it is found that the converged iteration decreases linearly with increasing BW and this decrease is periodic in nature as shown in Figure-5.

$$PAR(gn) = PAR_{\min} + \frac{(PAR_{\max} - PAR_{\min})}{NI} * gn$$

Where,

PAR- pitch adjusting rate for each generation

PAR_{min}- minimum pitch adjusting rate

PAR_{max}- maximum pitch adjusting rate

NI- number of solution vector generations

gn - generation number or converged iteration

$$BW(gn) = BW_{\max} - \frac{(BW_{\max} - BW_{\min})}{NI} * gn$$

Where,

BW - bandwidth for each generation

BW_{min}- minimum bandwidth

BW_{max}- maximum bandwidth

Figure5: Variation of PAR and BW versus generation number

We considered the total number of solution vector, HMS=50, the HMCR and PAR values were taken as 0.85 and 0.55 respectively. As shown in Table-1 HM was initialized with randomly generated solution vector within the bounds. Next a new harmony vector was improvised from possible values based on the three rules: memory consideration with 46.75% probability (0.85*0.55=46.75), pitch adjustment with 38.25% probability (0.85*0.45=38.25) and randomization with 15% probability (1-0.85=0.15) .The probability of finding global vector increases with number of searches. Finally, after 400 iterations, the HS gives near optimal harmony.

In our next study we compare harmony search with genetic algorithm and simulated annealing and result shows a fast convergence for HS than any other algorithm for same number of iteration. The HS algorithm incorporates the structure of current metaheuristic optimization algorithms. It preserves the history of past vectors (HM) similar to TABU search and is able to vary the adaption rate (HMCR) from the beginning to end of computations, which resemble simulated annealing. It also considers several vectors simultaneously in a manner similar to genetic algorithm. However, the major difference between GA and HS is that the latter generates new vector from all existing vectors (all harmony in HM), while GA generates new vector from only two of the existing vectors (parents).

Figure 3a: Above is observed (Blue) and inverted (Red) seismogram for noise free data and below is its velocity, density, thickness and impedance vatriation.

Figure 3b: Fitness error for noise free synthetic data, fitness value started at lowest value 0.38 and converged to a value 0.15

Figure 4a: Above is observed (Blue) and inverted (Red) seismogram for 30% noisy data and below is its velocity, density, thickness and impedance vatriation.

Figure 4b: Fitness error for 30 % noisy synthetic data, fitness value started at 0.78 and converged to a value 0.35

Conclusion

We have applied the recently developed HS meta-heuristic algorithm, that was conceptualized using musical process of searching for perfect state of harmony, to the inversion of band limited synthetic data in frequency ray-parameter domain assuming a 1D acoustic earth model. The use of plane wave reflectivity seismogram enables us to do large number of forward calculations very rapidly. We found that by performing a few HS inversion with varying pitch and bandwidth, we can approximately locate the best fit value in minimum iteration. However, it is found that HS converges much faster than simulated annealing or Genetic algorithm even if high iteration is given. This happens because the guidance of search has very low probability for randomization in Harmonic search. Further, HS algorithm can handle the complexity of problem very easily.

Table1: Values of inverted variables for different number of iterations

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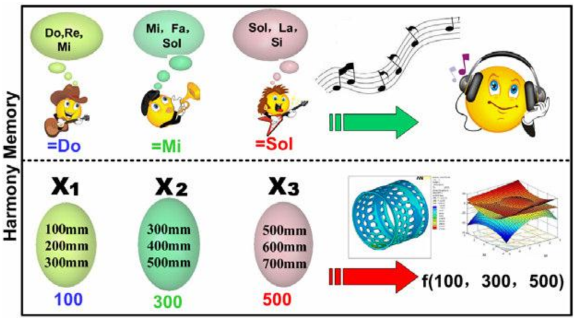

Figure 1: Analogy between music improvisation and optimization

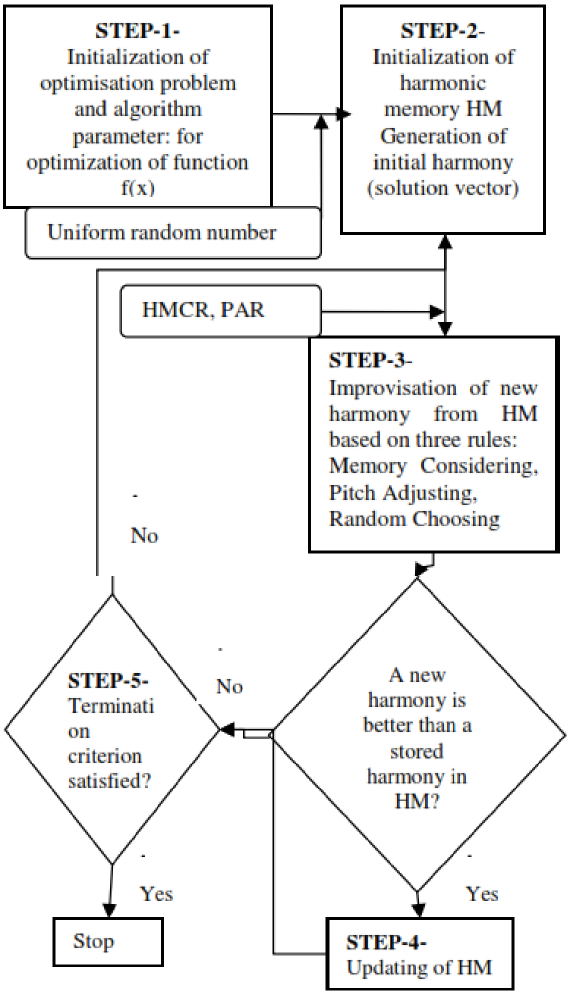

Figure 2: Flow chart of classical Harmony Search Algorithm.

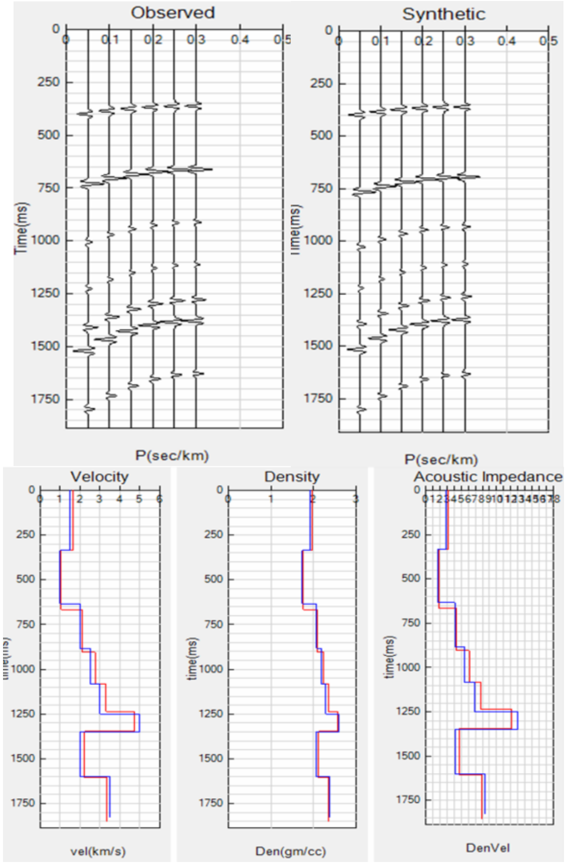

Figure 3a:Above is observed (Blue) and inverted (Red) seismogram for noise free data and below is its velocity, density, thickness and impedance vatriation.

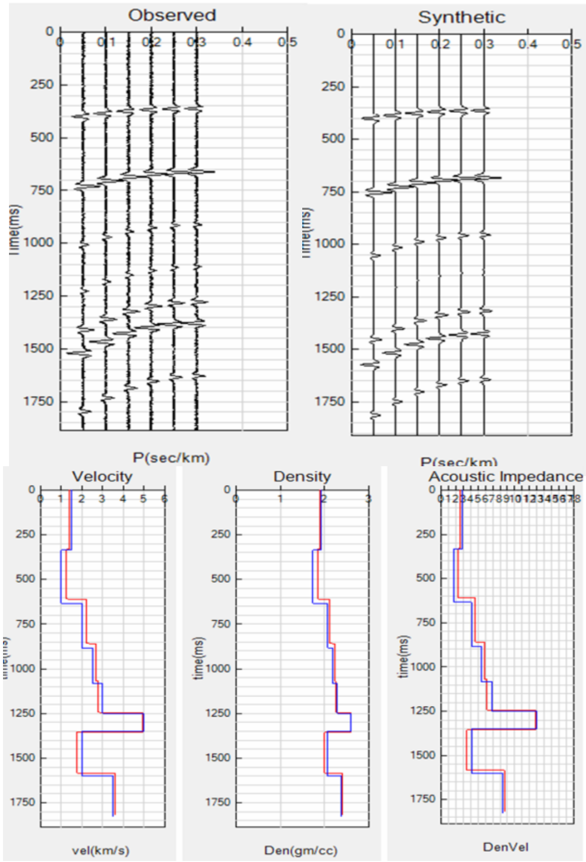

Figure 4a: Above is observed (Blue) and inverted (Red) seismogram for 30% noisy data and below is its velocity, density, thickness and impedance vatriation.

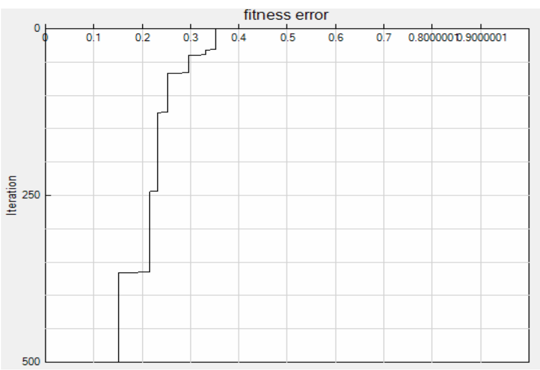

Figure 3b: Fitness error for noise free synthetic data, fitness value started at lowest value 0.38 and convergedto a value 0.15

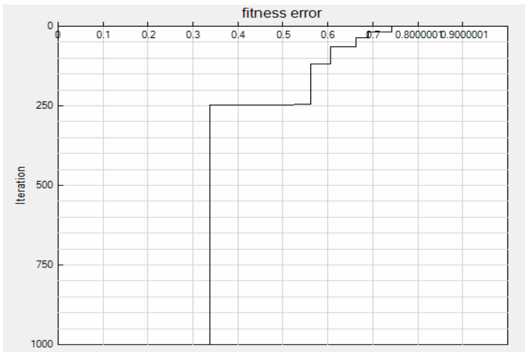

Figure 4b: Fitness error for 30% noisy synthetic data, fitness value started at 0.78 and converged to a value 0.35

Parameter	True value	Value after 50 iteration	Value after 100 iteration	Value after 300 iteration	Value after 400 iteration
Vel1(km/s)	1.5	1.95	1.81	1.72	1.51
Thick1(km)	0.25	.255	0.24	0.255	0.246
Density1(gm./cc)	1.9	2.06	2.02	1.99	1.92
Vel2	1	0.7	0.85	0.86	0.878
Thick2	0.15	0.096	0.1	0.19	0.134
Density2(gm./cc)	1.74	1.59	1.67	1.675	1.684
Vel3	2	1.586	2.22	2.147	1.861
Thick3	0.25	0.177	0.25	0.244	0.226
Density3(gm./cc)	2.07	1.95	2.124	2.11	2.03
Vel4	2.5	2.78	2.46	2.29	2.288
Thick4	0.25	0.295	0.266	0.24	0.235
Density4(gm./cc)	2.19	2.25	2.18	2.14	2.139
Vel5	3	3.56	3.246	2.852	2.837
Thick5	0.25	0.3	0.28	0.273	0.249
Density5(gm./cc)	2.289	2.39	2.335	2.261	2.26
Vel6	5	4.43	4.451	4.563	4.685
Thick6	0.25	0.25	0.19	0.299	0.24
Density6(gm./cc)	2.6	2.52	2.527	2.528	2.56
Vel7	2	2.6	2.32	1.965	2.012
Thick7	0.25	0.22	0.23	0.248	0.2488
Density7(gm./cc)	2.07	2.21	2.15	2.06	2.07
Vel8	3.5	3.36	3.6	3.36	3.468
Thick8	0.4	0.5	0.31	0.356	0.4
Density8(gm./cc)	2.379	2.356	2.396	2.356	2.374

Table1: Values of inverted variables for different number of iterations

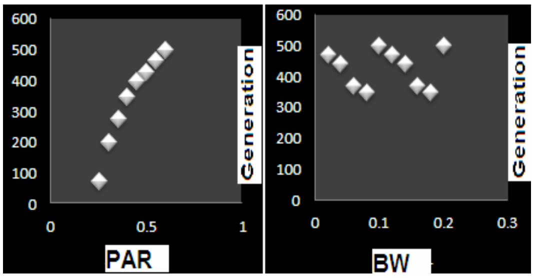

Figure5: Variation of PAR and BW versus generation number